Physics 522 - Quantum Optics
Lecture 16 Notes

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1 Introduction

We're currently in the middle of the "Quantum Optics of Photons" unit, which covers FQ5 through FQ8. To recap what we've seen in the previous chapters, let's look at the different methods we have to classify light.

1.1 Light Classification

Based on statistics alone, we have the following classifications:

Poissonian - perfectly coherent light. $I(t)$ is constant, and the distribution of photons in a given section of beam is given by the Poisson distribution ($\Delta n = \sqrt{\bar{n}}$).

Super-Poissonian - chaotic or incoherent light. $I(t)$ is not constant in time. The distribution of photons in a given section of beam is Poissonian in shape, but the standard deviation of n is larger than $\sqrt{\bar{n}}$

Sub-Poissonian - there is no classical description of this type of light. The photon distribution in a given section of beam is more tightly peaked around $\sqrt{\bar{n}}$ than in a strictly Poissonian distribution.

Another tool we have to classify light is the second-order correlation function, $g^{(2)}(\tau)$, which can be interpreted as proportional to the probability of measuring a photon at detector 1 at $t = t_0$, and detecting a photon at detector 2 at $t = t_0 + \tau$. Using this method of classification, we can divide light into the following categories:

Random - $g^{(2)}(0) = 1$. Random light has the interesting property that $g^{(2)}(\tau) = 1$ for all τ.

Bunched - $g^{(2)}(0) > 1$. For $g^{(2)}(0) > 1$, we conclude that there is a better than random probability of observing two photons simultaneously. Since you can obviously only measure a single photon once, this must mean that there was a "bunch" of photons traveling together that were split up, some sent to detector 1, and some sent to detector 2.

Anti-Bunched - $g^{(2)}(0) < 1$. In the perfectly antibunched case, the beam consists of a regular train of single photons. When this beam is incident on a beam splitter, there is only one photon sent to either detector at a given time, so the probability of observing two photons simultaneously is zero.

1.2 Coherent States and Squeezed Light

Recall that we can recast an electromagnetic field as a harmonic oscillator whose energy oscillates between two components: $p(t)$ and $q(t)$. This is analogous to the energy of a mass-spring oscillator being constantly converted between kinetic energy, $K \propto p^2$, and potential energy, $U \propto q^2$. The analogous quantities in the electromagnetic field are the electric field energy and magnetic field energies. Thus, the energy of an electromagnetic field oscillates between the electric and magnetic field components, similar to the oscillation of kinetic and potential energies in a mass-spring oscillator.

$$q(t) = \left(\frac{\varepsilon_0 V}{2\omega^2}\right)^{1/2} \mathcal{E}_0 \sin \omega t \propto \mathcal{E}_x(t)$$

$$p(t) = \left(\frac{\varepsilon_0 V}{2}\right)^{1/2} \mathcal{E}_0 \cos \omega t \propto \mathcal{B}_y(t)$$

$$E = \frac{1}{2}\left(p^2 + \omega^2 q^2\right)$$

To apply the quantum theory of the harmonic oscillator, we define the dimensionless quantities $X_1(t)$ and $X_2(t)$, called the field quadratures.

$$X_1(t) = \left(\frac{\omega}{2\hbar}\right)^{1/2} q(t)$$

$$X_2(t) = \left(\frac{1}{2\hbar\omega}\right)^{1/2} p(t)$$

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

States with $\Delta X_1 \Delta X_2 = 1/4$ are minimum uncertainty states.

Coherent states are minimum uncertainty states with some special properties. A coherent state can be written as a state vector, $|\alpha\rangle$, where α is a complex number.

$$\alpha = X_1 + iX_2 = |\alpha|\, e^{i\phi} = \left(X_1^2 + X_2^2\right)^{1/2} e^{i\phi}$$

This notation makes it evident that $|\alpha\rangle$ is a phasor in the complex plane of X_1 and X_2. Another important property to note is that the energy of the field, written as a harmonic oscillator, implies the following:

$$\hbar\omega\, |\alpha|^2 = \bar{n}\hbar\omega \quad \Rightarrow \quad \sqrt{\bar{n}} = |\alpha|$$

Squeezed states are coherent states for which the minimum uncertainty condition is still met, but $\Delta X_1 \neq \Delta X_2$. These states let you constrain the precise number of photons in a section of beam at the expense of the phase relationship between the photons or vice versa.

2 Photon Number States

Photon number states are states for which the number of photons in a given mode is fixed. These photon nubmer states are analogous to harmonic oscillator Hamiltonian eigenstates.

$$\hat{H}\,|n\rangle = E_n\,|n\rangle = (n+1/2)\hbar\omega\,|n\rangle$$

Just like the quantum harmonic oscillator has raising and lowering operators, photon number states have raising and lowering operators which satisfy the following conditions:

$$\hat{a}^\dagger\,|n\rangle = (n+1)^{1/2}\,|n+1\rangle$$
$$\hat{a}\,|n\rangle = (n)^{1/2}\,|n-1\rangle$$
$$\hat{a}^\dagger\hat{a}\,|n\rangle = n\,|n\rangle$$
$$\hat{a}\,|0\rangle = 0$$

When discussing the raising and lowering operators of the quantum harmonic oscillator, you discover that it is possible to write them in terms of the position and momentum of the harmonic oscillator (p and q). Again, we find an analogous relationship in the context of quantum optics.

$$\hat{X}_1 = \frac{1}{2}(\hat{a}^\dagger + \hat{a})$$
$$\hat{X}_2 = \frac{1}{2}i(\hat{a}^\dagger - \hat{a})$$

2.1 Coherent States

With the formalism of photon number states we can revisit coherent states and hope to shed some light on their properties. We can write a general coherent state $|\alpha\rangle$ in the following way:

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}\,|n\rangle$$

Applying the raising and lowering operators to $|\alpha\rangle$ makes the operators hit the linear combination of number states, providing us with the following properties:

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

$$\langle\alpha\mid\hat{a}^\dagger = \langle\alpha\mid\alpha^*$$

$$\langle\alpha\left|\hat{n}\right|\alpha\rangle = \langle\alpha\left|\hat{a}^\dagger\hat{a}\right|\alpha\rangle = \alpha^*\alpha = |\alpha|^2$$

It is important to note that $|\alpha\rangle$ is not an eigenstate of the Hamiltonian, so making an observation on the state $|\alpha\rangle$ will collapse $|\alpha\rangle$ to a Hamiltonian eigenstate, namely one of the photon number states, $|n\rangle$. Recalling that $|\alpha\rangle$ is a linear combination of the photon number states, we can determine the probability of observing state $|n\rangle$ by calculating the modulus squared of the coefficient of $|n\rangle$ in the linear combination of photon number states that $|\alpha\rangle$ is comprised of.

$$|\alpha\rangle = \sum_n c_n\,|n\rangle$$

$$\mathcal{P}(n) = |c_n|^2 = |\langle n\mid\alpha\rangle|^2$$

$$c_n = e^{-|\alpha|^2/2}\frac{\alpha^n}{(n!)^{1/2}}$$

$$\mathcal{P}(n) = |c_n|^2 = e^{-|\alpha|^2}\frac{(|\alpha|^2)^n}{n!} = \frac{\bar{n}^n}{n!}e^{-\bar{n}}$$

We see that we recover our old friend the Poisson distribution, furthering the analogy of coherent states as the quantum analogs of perfectly coherent beams of light.

2.2 Quantum Theory of HBT Experiments

To discuss HBT experiments, let's set up our apparatus. We have two input ports, 1 and 2, which feed into a 50:50 beam splitter. The beam splitter outputs are fed to detectors, 3 and 4. The detector outputs are fed to a START/STOP timer which starts a stopwatch when a photon is measured at the START detector, and stops the stopwatch when a photon is measured at the STOP detector.

A beam of prepared light is incident at detector 1, while detector 2 is exposed to the vacuum field. With these definitions, the second-order correlation function takes the following form:

$$g^{(2)}(\tau) = \frac{\langle n_3(t) n_4(t+\tau)\rangle}{\langle n_3(t)\rangle\langle n_4(t+\tau)\rangle}$$

By rewriting n_3 and n_4 in terms of the raising and lowering operators and employing other identities, we can greatly simplify the expression for $g^{(2)}(\tau)$.

$$g^{(2)}(0) = \frac{\hat{n}(\hat{n}-1)}{\hat{n}^2} = \frac{n(n-1)}{n^2}$$