**Trapped Ion Quantum Computation**

**With Ytterbium**

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Physics 522 Quantum Optics

**1) Quantum Computation**

Quantum Computation is a rapidly growing field which has the potential to offer vast improvements in certain fields of computation From a computer science perspective, quantum computation is exciting because it has been shown that certain “hard” problems (required resources scale exponentially with complexity) in classical computing can be reduced to relatively “easy” problems (required resources scale polynomially with complexity) when performed with a quantum algorithm. To give a good example of the difference between hard and easy, I’ll borrow and example given by John Preskill from Caltech. Factoring a product of two prime numbers ranks as “hard” when being performed on a classical computer, yet while solving the equivalent problem using the quantum version, Shor’s Algorithm is reduced to an “easy” problem. To factor a product of two primes which is 193 digits long, a classical computer operating at 2.2GHz would take about 30 CPU years. A quantum computer of similar specs, operating at 2.2GHz, would require 0.1 seconds. If the problem was scaled up to factoring a 500 digit number, it would take the classical computer 1012 years, the quantum computer would be able to solve the same problem in only 2s! While only a few notable algorithms have been demonstrated so far, the field is of great interest because no one has found out exactly how far the class of quantum “easy” problems extends in complexity space.

Before going any further it is probably worth addressing the question “what is a quantum computer?” While there are many physical systems which researchers are trying to implement to build a quantum computer, none of them uniquely describe what a quantum computer *is.* The most commonly accepted definition of a quantum computer is given by the DiVincenzo Criteria which state that a quantum computer must have:

1. Physically Scalable, Well defined Qubits
2. The ability to initialize to a well characterized fiducial state
3. Relatively long coherence times
4. Implementation of a Universal Set of quantum Gates
5. Accurate measurement of resulting qubit state

A well-defined Qubit implies an effective two level system: a system which is sufficiently understood such that any other relevant states of the system besides the two qubit states can be effectively isolated or appropriately addressed. The physically scalable part of the first postulate is a bit vague and is one of the last hinders to a building a computationally useful quantum computer. The best definition I could give would be a system which does not have a fundamental limit to the number of qubits that can be entangled. Also, as a physical necessity, the resources needed in its implementation must not scale exponentially or in any unreasonably manor as the number of qubits increases. The ability to initialize to a fiducial state also known as simply state preparation is necessary due to requirements imposed by several quantum algorithms. This criterion is not as stringent as the others because some quantum algorithms or being devised which can accurately be performed with the qubits starting in a range of states, for instance not explicitly the ground state. The third criterion is somewhat self-explanatory, the coherence times of the qubit must be relatively long in comparison to the duration of time it takes to perform quantum gates on the system such that a complete an entire algorithm without decoherence. For the fourth criterion, several groups in the 90’s demonstrated that combinations of single qubit rotations or single qubit gates, and a 2-qubit entangling gate was sufficient to produce a family of universal quantum gates~ eg much like how any classical algorithm can be performed with combinations of “NOT” and “AND” gates, any quantum computation can be constructed out of single qubit rotations and a 2-bit entangling operation such as the C-NOT gate. Lastly, it is necessary to perform a high fidelity state readout at the end of any quantum algorithm to observe the results. This might again seem trivial but several experimental constraints on some systems can make this component very challenging. Furthermore sometimes it is beneficial to have the ability to perform a non-demolishing state read out, eg if you measure state 0 or state 1, the qubit will remain or decay back into the same state following measurement. Several physical systems have been demonstrated as possible qubits such as superconducting flux qubits in Josephson junctions, polarization qubits with photons, and nuclear spin in NMR. For the rest of this paper I shall be discussing the use of the hyperfine states of electromagnetically confined ions for quantum computation.

**2) Trapping Ions**

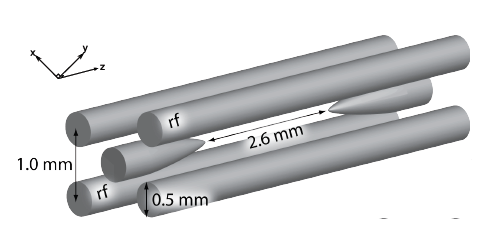
Ion traps are currently the most advanced system being utilized for quantum computation, having successfully satisfying all of DiVincenzo’s criteria sans demonstration of a fully scalable system, as of 2011 researchers had successfully entangled 14 trapped ions in a trap. While trapped ion qubits do have several intrinsic properties that give them advantages over rival systems, the main reason why trapped ions are so far is because many of the relevant techniques for trapping and controlling ion’s down to a quantum regime had been previously developed in other pursuits. Wolfgang Paul pioneered the development of what would become known as the RF-Paul trap for use as a mass spectrometer in the 40’s and 50’s, while there are other methods for confining charged particles the RF Paul trap serves as the workhorse for the trapped ion community. In the 70’s and 80’s a great deal of work was done by David Wineland et all in the development of Laser Cooling techniques which are essential in providing the necessary control for trapped ions as useful qubits. With these and several other components in place already, when Ignacio Cirac and Peter Zoller published their famous paper detailing a protocol for performing a C-NOT gate with trapped ions in 1994 it took only a year for David Wineland and Chris Monroe to demonstrate it successfully at NIST in 1995. I think it is worth spending a little bit of time going over the basic of how an ion trap, specifically the RF-Paul trap, works before going further into quantum computation.

Earnshaw’s theorem states that it is impossible to confine a charged particle in space with static potentials alone. This can be seen easily from Gauss’s Law in the absence of any charge,

While it is possible to have a confining potential in one direction there will always be at least one unbound direction, eg there are no stable equilibrium points on saddle points. The RF Paul trap overcomes this obstacle by adding an oscillating RF potential of a quadruoplar shape. Quadrupolar potentials have an (r2) spatial dependence near their centers giving rise to a linear restoring force which will be desirable for several reasons later on.

The example trap show in figure 1 demonstrates a common four rod geometry where all four rods are roughly parallel, straight, and evenly spaced. An oscillating RF potential is applied to two opposing rods, while the two remaining rods are grounded. In addition there are two pointed end-caps to which a DC potential is applied, providing confinement in the axial (z) direction of the trap. This gives rise to a potential of the form

Figure I – RF Paul Trap



The equations of motion of motion for such a trapping geometry are separable and soluble via newton’s laws. The z or axial direction is seen to be simply a harmonic potential, while the x and y (transverse) directions turn out to be slightly more complicated.

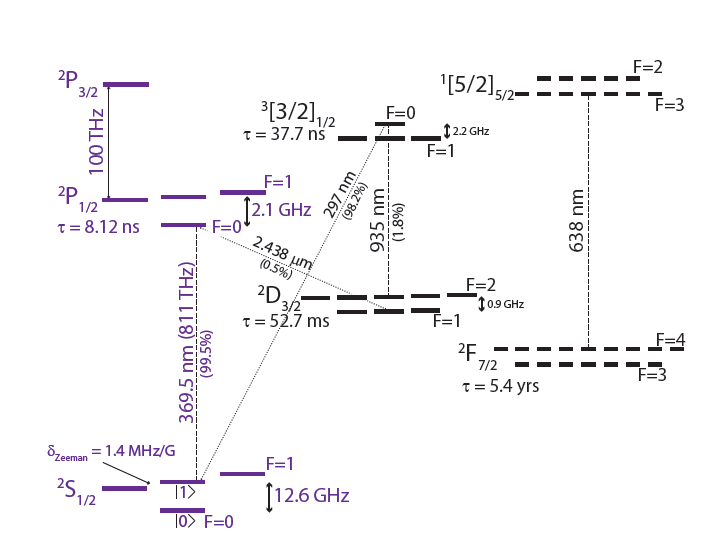
Fortunately, upon inspection the transverse equations can be cast as Mathieu equations, which are a type of ODE which can be solved using Floquet theory for analyzing periodic systems, which has well-studied solutions.

There is a great deal of physics involved in deriving the equations of motion of a trapped ion and such a potential, in order to give rise to the stable equations of motion shown several approximations and steps have been made which are beyond the scope of this paper. The ‘a’ parameter is proportional to the DC component of the potential U, while the ‘q’ parameter is proportional to the RF potential V, for typical 4 rod traps of the geometry shown in figure 1, it turns out that the RF effects dominate, yielding typical values where (a­ << q << 1). Under these conditions, I hope to convey an approximate picture of the relevant physics being described; the equations of motion represent large slow oscillations at what is called the secular frequency ‘ω’, and smaller, faster oscillations at the RF frequency ‘Ω’. Furthermore returning to the axial equations, if (d > R) it can be seen that the axial frequency is always the lowest of the three frequencies. When more than a single ion is loaded in a Linear 4 rod RF-Paul trap, under sufficient constraints the ions will be seen to line up in a straight line. The motion of the ions can then be constructed via excitations of the various normal modes of the system related to these trapping frequencies.

**3) Ytterbium**

One of the most popular species used in trapped ion quantum computation is Ytterbium, a group two rare earth element. Specifically singly ionized (Yb171+) with nuclear spin of ½ is used, giving rise to a hyperfine splitting of electronic states. The , ground states are used as our ‘0’ and ‘1’, ‘up’ and ‘down’ states. These two hyperfine ground states have an energy splitting of 12.6GHz and are insensitive to first order magnetic field effects and only slightly susceptible to B2 effects. These states are very desirable for their relatively long coherence times T­2~10s.

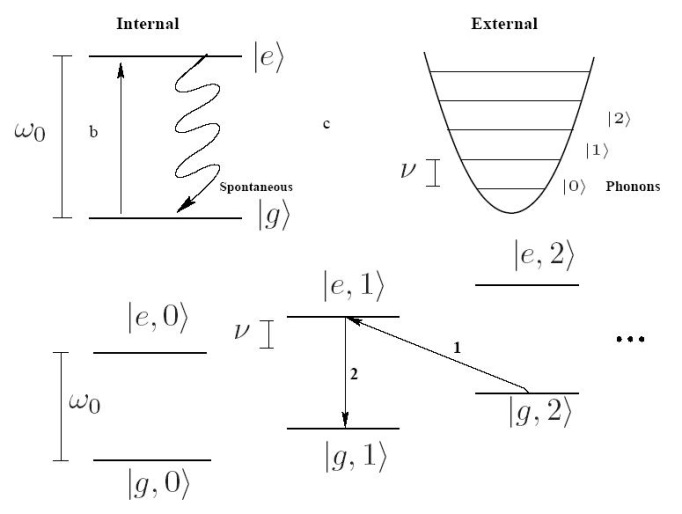
Figure II – YB171+ Energy Levels



To trap Yb171+, an atomic beam of Yb171 is directed towards the center of the trapping potential. This is typically generated by a simple ceramic tube filled with solid Yb which is heated by running current through a tungsten wire wrapped around the outside of the cylinder. A 399nm directed at the center of the trap excites the neutral atom to the p shell. A second laser which is used for several purposes in the experiments, the 369nm “cooling” beam, has sufficient energy to photo-ionize the excited electron into the continuum. For sufficiently strong traps the ion will be bound, but in an extremely excited state, Room temperatures correspond to the <n> ~ 10^5 state of the trap SHO potential. In order to be imaged or used in any subsequent experiments it is necessary to cool try ion, (in some experiments it is actually necessary to reach the ground state!)

The ions are initially cooled via Doppler Laser Cooling via the 369 beam. The basic idea is to shine a laser on the ions with a frequency red detuned from a closed cycling transition. Due to the thermal motion of the ion, in the ions frame the cooling beam will appear Doppler shifted. If the ion is moving antiparallel, towards the incoming beam the ion will see light which has a blue Doppler shift that is closer to the resonant scattering transition and hence scatter lots of light. If the ion is moving away, traveling in the same direction as the incident beam, the incident light will appear even further red detuned from resonance and the ion will scatter much less light. With appropriate detuning the ion can be thought of essentially scattering only when it is moving towards the beam. When the atom absorbs a photon it gains an h-bar k momentum kick in the direction of the beam, if the atom is moving towards the beam this momentum is in the opposite direction of its own velocity and results in the atom losing h-bar k momentum. Following absorption the ion will quickly undergo spontaneous emission via an electric dipole transition causing the atom emit a photon in a random direction resulting in the ion getting an h-bar k momentum kick in the opposite direction. It may appear that the end result is no change in the momentum, but since the spontaneous emission is in a random direction the momentum kicks will average out to zero while the absorption momentum kicks will always be opposite to its motion. Due to the high scattering rate between the s-p states, the ion can be cooled to a limit of <n> ~ 5 oscillator state. To cool to even lower states Doppler side band cooling can be implemented. Side band cooling works by applying sidebands to the cooling beam at the trap secular frequency. This can allow an ion in a ground/excited state to absorb/emit a photon into the excited/ground state with the n-1 oscillator state, eg removing a phonon with the transition. Doppler Sideband cooling can be used to reliably cool an ion down to the n=0 state of motion.

Figure IV – Sideband Cooling



**4) Quantum Gates**

As mentioned before a universal set of quantum gates can be constructed via a combination of a single qubit rotation gate and a 2 qubit entangling gate. The Hamiltonian of a trapped ion interacting with a laser field can be shown to be

Where is the Lamb-Dicke parameter (measure of how much light couples to ions motion), is the trap secular frequency, is the effective Rabi rate (dependent on power of laser driving transitions), are the raising lowering spin operators, and are the creation and annihilation operators for the motional harmonic oscillator states. In the Lamb-Dicke regime we have the following Hamiltonian which we will study for three cases of the lasers detuning to the ions resonance frequency

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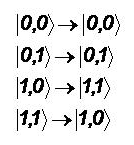
These Hamiltonians can give rise to the single qubit rotation operators,

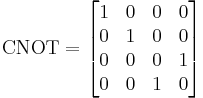
*Single Qubit Rotations*

Single qubit gates can be simply performed by performing Rabi oscillations with the desired phase using the single qubit rotation operators. The two-qubit entangling gate which is also required for a quantum computer is generally much more complicated. The first quantum gate proposed for trapped ions was the ‘Cirac-Zoller’ gate. The Cirac-Zoller gate describes a protocol for implementing a CNOT gate on trapped ions. The gate works as follows:

1. 1 The internal state of a control ion is mapped onto the motion of an ion string
2. The state of the target ion is flipped conditioned on the motional state of the string
3. Motion of ion string is mapped back to control ion state

\*Result – Flips target bit if control bit is in a certain state

Figures- V, VI, - C-Not Logic tables



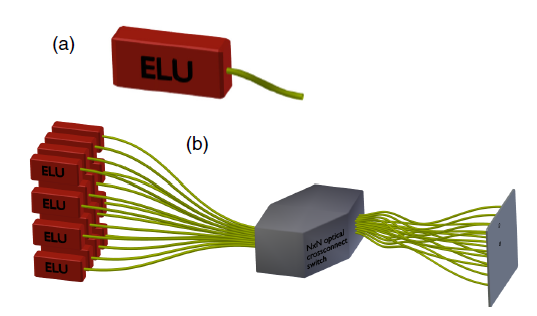
One possible pulse sequence which can be implemented to perform a CNOT gate via a string of pulse ions is, , where upper index +,C refer to the raising and carrier transitions, and the lower indices c and t represent target and control ion.



**5) Future**

As of yet, all of Divincenzo’s criteria have been satisfied for trapped ion systems, with the exception of scalability. No quantum system has been proven to be efficiently scalable to the size required for performing useful calculation ~ 1000 qubits. There are two popular plans which hope to overcome several challenges to allow for trapped ion systems to scale to large numbers of qubits. One which is being researched by several groups is the MUSIQC program (Modular universal scalable ion trap quantum computer. The goal of this design is to create an array of qubits in an ion trap which can be efficiently handled with high accuracy. Then make several copies of this modular device and provide a way to entangle the state of the modules to a flying photonic qubit which can be coupled to a fiber. The photonic qubits can then be interfered and entangled in the proper ways to allow for performing relevant quantum gates. A second proposal being worked on by several other research groups is attempts building large scale arrays of traps with ion shutting routes running between linear trapping zones. The idea being again find some desirable array size of ions to work with, perform gates locally on the small arrays then shuttle an entangled ion from array to array to pass on information. This approach is fraught with many difficulties, specifically undesirable effects due to motional decoherence due to the large numbers of ions and their relative closeness to the trap electrodes. Hopefully in the future many advances will be made and finally allow a full scale quantum computer to come to fruition.

Figure VII – MUSIQC proposed design



**References**

1. *Steven Olmschenk.. “QUANTUM TELEPORTATION BETWEEN DISTANT MATTER QUBITS” Doctoral Thesis. University of Maryland. (2009)*
2. *David Hayes. “Remote and Local Entanglement of Ions using Photons and Phonons”. Doctoral Thesis. University of Maryland. (2012)*
3. *Johnathan Mizrahi. “ULTRAFAST CONTROL OF SPIN AND MOTION IN TRAPPED IONS”. Doctoral Thesis. University of Maryland. (2013)*
4. *Timothy Andrew Manning, “QUANTUM INFORMATION PROCESSING WITH TRAPPED ION CHAINS”. Doctoral Thesis. University of Maryland. (2014)*
5. *Chris Monroe, et all. “Large-scale modular quantum-computer architecture with atomic memory and photonic interconnects”. PHYSICAL REVIEW A 89, 022317 (2014)*
6. *DiVincenzo, David “The Physical Implementation of Quantum Computation”. ARXIV.* *arXiv:quant-ph/0002077v3 13 Apr 2000 (2008)*
7. *J I Cirac, P. Zoller. “Quantum Computations with Cold Trapped Ions”. Physics Review Letters. Volume 74 . Issue 20. May 15 (1994)*
8. *H. H¨affner, C. F. Roos, R. Blatt, “Quantum computing with trapped ions”. ARXIV.* *arXiv:0809.4368v1 [quant-ph] 25 Sep 2008 (2008)*