

# ***Boson Sampling and Quantum Assisted Computation***

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ECE 695 Quantum Optics and Photonics

## **1. Introduction: quantum computing and its current limitations**

The ultimate continuation of Moore's Law lies in the realization of quantum computing. The power of quantum computing is derived from the usage of quantum superposition, allowing a quantum bit (qubit) to take the value 0, 1 or any combination therein simultaneously. This allows a quantum computer with  $n$  qubits to represent a superposition of *all* of the possible  $2^n$  bit combinations simultaneously, in contrast to traditional computers which can only exist in *one* of the  $2^n$  combinations at any one time. Quantum computation is achieved by applying quantum logic gates which manipulate the quantum state of the qubits. At the end of the manipulation, a measurement is made, causing the wavefunctions of the qubits to collapse and providing  $2^n$  standard data bits. Qubits and quantum computation can be achieved through several potential methods including spin-states (spintronics) [1], topological insulators [2], superconducting circuits [3], ion traps [4], and single photon optics [5]. We will focus on quantum optics and single photon related quantum computation in this report.

The lure of quantum computers and their massive computation potential have kept research in this field vibrant with many important algorithms having been developed along with experimental demonstration of qubit manipulation. However, many challenges towards the realization of a working quantum computer remain such as de-coherence and many-body entanglement [6]. Currently, the record for entangled states is at 14 qubits, but with limited coherence time which reduces the number of operations that can be performed [7]. Methods of error correction which combat de-coherence allowing for greatly enhanced coherence time have recently been observed in solid-state systems utilizing 3 qubits [8]. These works are seminal in forwarding quantum computing towards practical uses, but most scientists believe that fully quantum computers are many decades away from practical applications.

## **2. Boson sampling**

As a result of the difficulties associated with constructing a fully quantum computer, many scientists have suggested alternative methods for quantum-assisted computation. One such method is boson sampling which calculates a system's response based on single photon interference, but does not enable universal quantum computation [9]. Boson sampling can be thought of as a quantum equivalent of the Galton board, as shown in Fig. 1(left), or as an analog computer. Here, the Galton board represents a simple computer which samples the binomial distribution. By modifying the size and order of the pegs (input variables) the board samples the probability distribution of the system being modeled by the variables (output). In the case of boson sampling, the ordering of the pegs is replaced with beamsplitters and delay lines and the identical balls are replaced with non-interacting identical bosons (photons are most typically used because they are non-interacting, which is important to insure unitary matrix transformations), see Fig 2. The resulting system is governed by quantum photon statistics and is used to model the provability distribution of an arbitrary system.

Initial applications of Boson computers have been focused on unitary matrix transformations, specifically determining the permanent of a sub-matrix (which is similar to the determinant, see Eq. 1.1, 1.2). This is particularly important when considering large linear optical networks with many

components. Such a system cannot be accurately modeled by classical methods when the number of variables (inputs, outputs, and components) is greater than roughly 20. This is because calculating the permanents grows exponentially with the number of variables [9-11]. However, the modeling of the system using a boson sampling computer is less than exponential complexity. Thus, for systems that are adequately large, boson sampling can outperform traditional computational modeling.

$$Perm(A) = \sum_{\sigma \in S_n} \prod_{i=1}^n a_{i\sigma(i)} \quad (1.1)$$

$$Perm \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad + bc \quad (1.2)$$

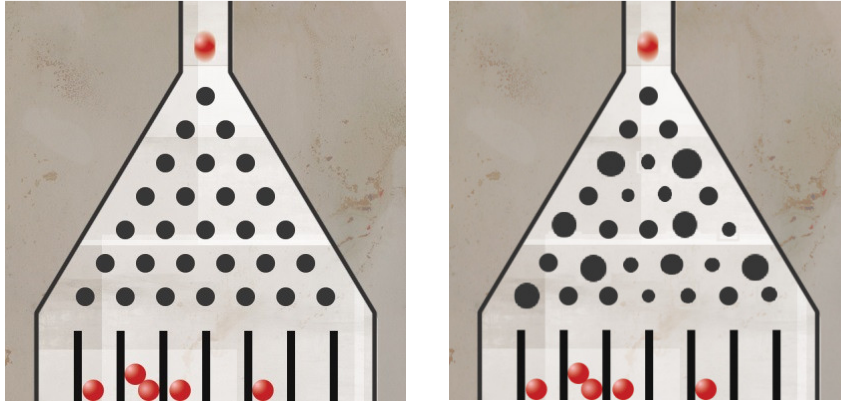


Fig. 1 (left) Traditional Galton board whose output (collection of balls at the bottom) forms a binomial probability distribution.(right) Modified Galton board to simulate a specific system. The output of this board represents the probability distribution of the system being modeled.

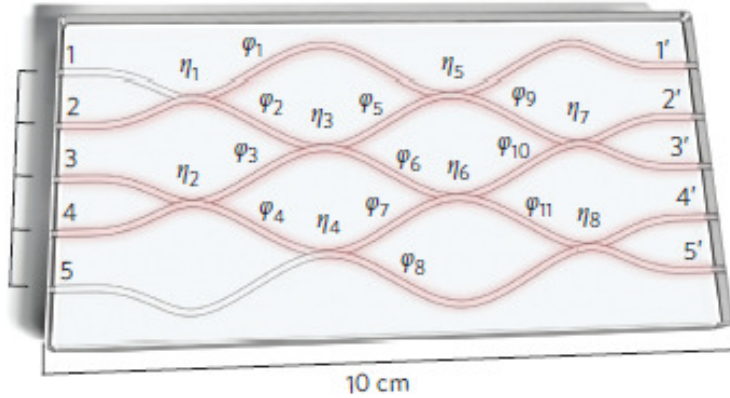


Fig. 2 Illustration of a boson sampling network which numerically resembles a Galton board for photons. The beamsplitters, represented as  $\eta$ , and delay lines, represented by  $\phi$ , model a desired system. This system allows the user to probe the probability distribution of the system being modeled through successive experimentation [10].

Boson sampling is also very important for modeling other quantum phenomenon, an extremely hot topic in computer science today. Because of the non-trivial nature of many quantum processes, they are inherently difficult to model classically. Boson sampling can efficiently model such processes by essentially letting the physics do the work, and we just observe the output. Before we discuss specific examples of boson sampling we first should first review the effects of single and two-photon interference.

After this, examples of working boson sampling machines will be presented followed by an outlook for this technology. <sup>†</sup>

#### a. Single and two-photon interference

Single photon interference can be analyzed by considering a photon incident upon a 50-50 beam splitter, see Fig 3. The classical beam splitter with a single input beam would be written such that  $A_2 = rA_1$  and  $A_3 = tA_1$ . However, when we consider the quantum mechanical view of this situation (i.e.  $\hat{a}_2 = r\hat{a}_1$  and  $\hat{a}_3 = t\hat{a}_1$ ), the commutation relations are not satisfied. We must consider a second input as the quantum vacuum state, not pure zero. With this inclusion we can write the quantum mechanical relations of the beamsplitter as  $\hat{a}_2 = r\hat{a}_1 + t'\hat{a}_0$  and  $\hat{a}_3 = t\hat{a}_1 + r'\hat{a}_0$ . For a 50-50 beamsplitter, the reflected beam has an additional  $\pi/2$  phase shift such that  $\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1)$  and  $\hat{a}_3 = \frac{1}{\sqrt{2}}(i\hat{a}_0 + \hat{a}_1)$ . If we consider that the input state of the beamsplitter is  $|0\rangle_0 |1\rangle_1 = \hat{a}_1^\dagger |0\rangle_0 |0\rangle_1$  we find that the output state is given as:

$$\begin{aligned} |0\rangle_0 |1\rangle_1 &= \frac{1}{\sqrt{2}}(i\hat{a}_2^\dagger + \hat{a}_3^\dagger)|0\rangle_2 |0\rangle_3 \\ &= \frac{1}{\sqrt{2}}(i|1\rangle_2 |0\rangle_3 + |0\rangle_2 |1\rangle_3) \end{aligned} \quad (1.3)$$

From these results we see that there can be no coincidence counts at detectors D1 and D2 in Fig. 3. This is exactly the results we would expect, it says that a photon cannot be divided by the beam splitter. In fact, the detectors would both record count probabilities of  $\frac{1}{2}$ . This problem of the 50-50 beam splitter is so well understood that is used a proof of single photon generation (i.e. HBT interferometer and  $g^{(2)}$  measurement) [12].

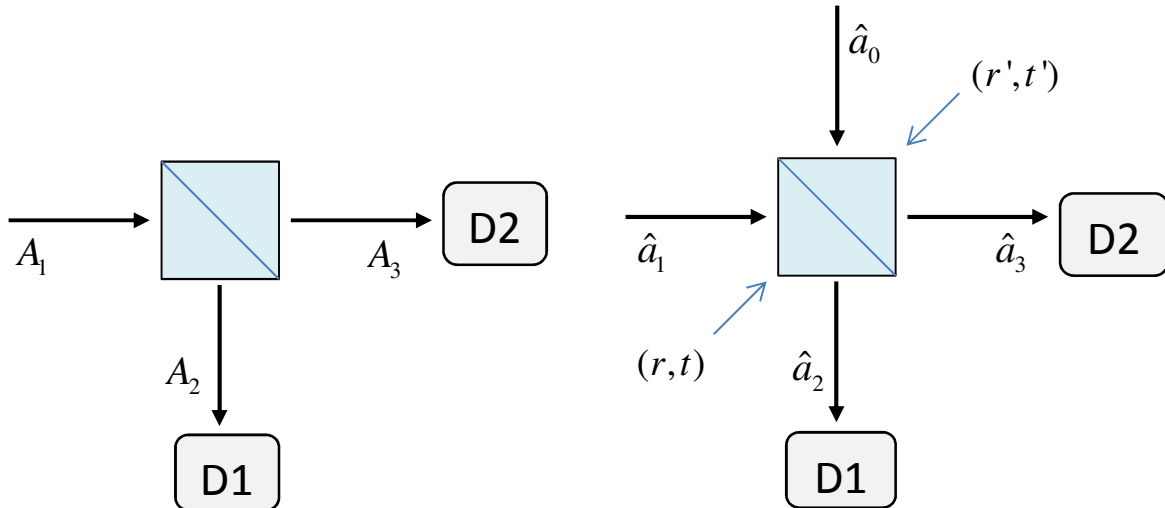


Fig. 3 Illustration of single photon interference experiment with a single 50-50 beamsplitter. (left) classical field amplitudes where only one side of the beam splitter contains an excitation field. (right) quantum view of the beam splitter where the second input port is considered as the quantum vacuum.

Two photon interference is a non-trivial affair which requires quantum analysis to explain. Let us consider a 50-50 beamsplitter with two identical photons incident as shown in Fig. 4. The four potential

output combinations are also listed in Fig. 4. Here we consider an input state  $|1\rangle_0 |1\rangle_1 = \hat{a}_0^\dagger \hat{a}_1^\dagger |0\rangle_0 |0\rangle_1$ . Similarly, we can write the output of the beam splitter as:

$$\begin{aligned} |1\rangle_0 |1\rangle_1 &= \frac{1}{2} (\hat{a}_2^\dagger + i\hat{a}_3^\dagger) (\hat{a}_2^\dagger + \hat{a}_3^\dagger) |0\rangle_2 |0\rangle_3 \\ &= (i\hat{a}_2^\dagger \hat{a}_2^\dagger + \hat{a}_2^\dagger \hat{a}_3^\dagger - \hat{a}_3^\dagger \hat{a}_2^\dagger + \hat{a}_3^\dagger \hat{a}_3^\dagger) |0\rangle_2 |0\rangle_3 \\ &= \frac{i}{\sqrt{2}} (|2\rangle_2 |0\rangle_3 + |0\rangle_2 |2\rangle_3) \end{aligned} \quad (1.4)$$

What we see from this analysis is that the situations where both photons are transmitted or both photons are reflected cancel exactly. This causes both photons to appear in the same pathway such that there should still be no coincidence counts. However, unlike the single photon effect, the results can be likened to the wave-like nature of light and resulting interference. Quantitatively we can understand why these situations cancel by invoking Feynman's rule: the probability of an outcome with multiple indistinguishable results is determined by the square of the modulus of the individual process probability amplitudes. We recall that the reflection results in a  $\pi/2$  (or  $i$ ) phase shift, and find the probability as:

$$P_{11} = |A_T A_T + A_R A_R|^2 = \left| \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \frac{i}{\sqrt{2}} \right|^2 = 0 \quad (1.5)$$

This situation has been experimentally demonstrated by Hong, Ou, and Mandel as well as many others since including with quasi particles such as single plasmons [13].

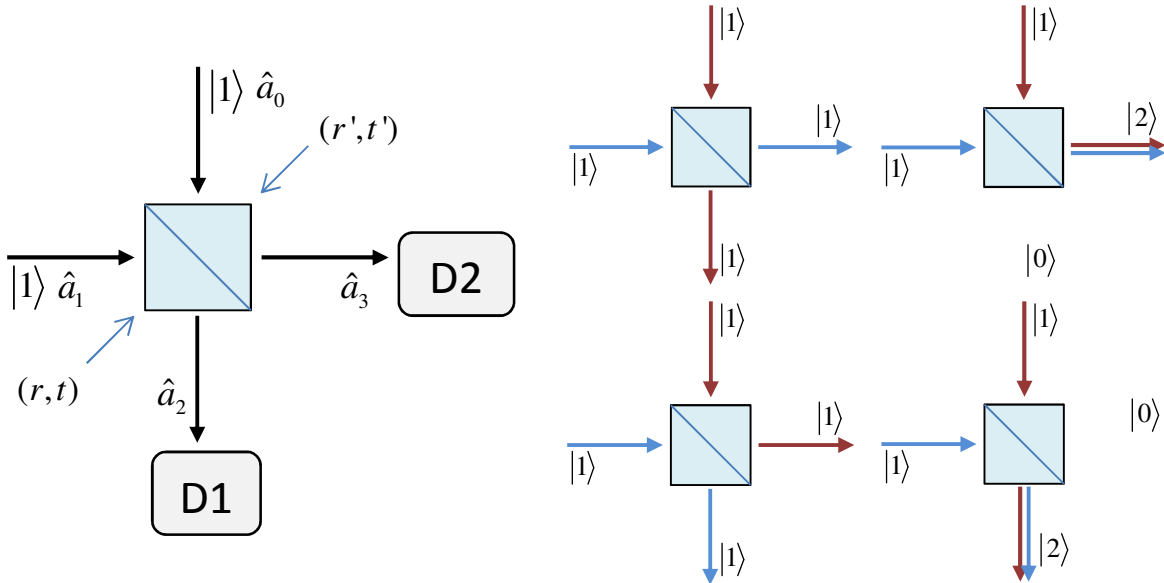


Fig. 4 (left) Illustration of Hong-Ou-Mandel interferometer for two photon interference effects. (right) The four potential outcomes of the two-photon interference effect.

For realistic boson sampling machines, various distributions of photons are input into the system with many inputs and many outputs (see Fig. 2). These are described by a state such as  $|1201\rangle$  which corresponds to 1 photon insert at input 1, 2 at input 2, and 1 at input 4. If we start with a state  $|I\rangle = |i_1 i_2 i_3 \dots i_m\rangle$ , the probability of finding a given state  $|O\rangle = |j_1 j_2 j_3 \dots j_m\rangle$  can be shown to be [9]:

$$P_{I,O} = \frac{|Per(U_{I,O})|^2}{i_1! i_2! i_3! \dots i_m! j_1! j_2! j_3! \dots j_m!} \quad (1.6)$$

The matrix  $U_{I,O}$  is defined as the elements in the  $m \times m$  matrix corresponding to the states  $i$  (columns of U) and  $j$  (rows of U) where the input and output have photons inserted (see following example for  $|I\rangle = |110\rangle$  and  $|O\rangle = |011\rangle$ ) [10]. Note that the minus sign in Eq. 1.8 comes from minor diagonal terms in the U matrix, which are imaginary (represents reflections).

$$U = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \rightarrow U_I = \begin{bmatrix} a & b \\ d & e \\ g & h \end{bmatrix} \rightarrow U_{I,O} = \begin{bmatrix} d & e \\ g & h \end{bmatrix} \quad (1.7)$$

$$P_{I,O} = |dh - eg|^2 \quad (1.8)$$

### b. Examples of boson sampling

Boson computers are able to determine the matrix permanents by studying the propagation of single photons through a representative optical network. As we mentioned before, this network is composed of two main components, beamsplitters and delay lines, see Fig 2. Single photons are inserted into various input ports of the system and their propagation through the system is studied over time. A photon that enters in port 1 may exit at port 5, with a given probability. The probability of finding photons in a given mode is determined by the permanent of the matrix.

Boson sampling has been experimentally demonstrated by four groups so far. We will discuss two here and the other works can be found through these references [14, 15]. While the works are all quantitatively similar (and were coordinated to be published simultaneously) they do have some small differences. The first result to discuss is from Philip Walther et al who demonstrated a  $M = 5$  input and output (arbitrary 25 element matrix) boson sampling machine using three input photons. Several independent systems were designed using direct laser writing and their probability distributions were studied with the boson sampling technique. An example of the boson sampling chip is shown in Fig.5 (left) with recorded output in Fig. 5(right). The experimental setup is shown in Fig. 6 [10]. After the probabilities of the various output modes were recorded the permanents of the mode combinations were fitted to experimentally determine the unitary matrix U.

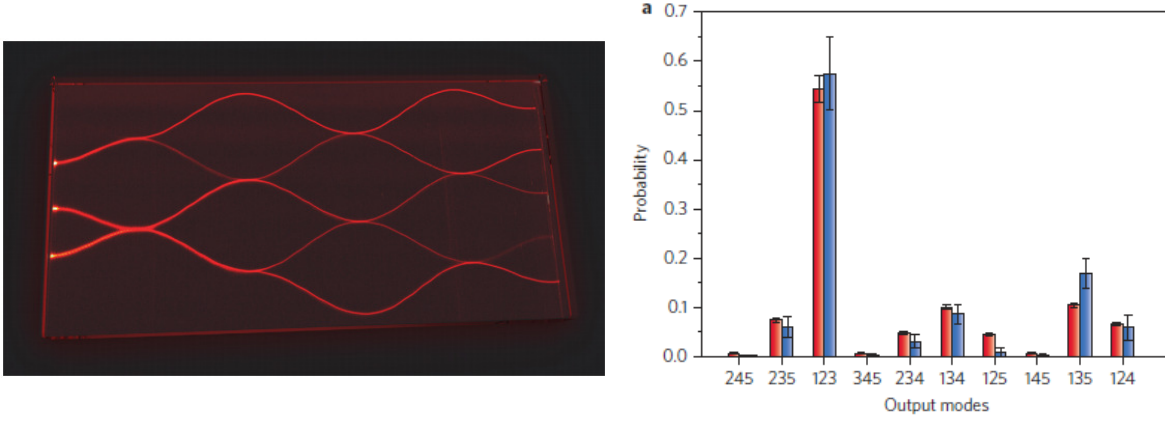


Figure 5 (left) Time averaged fluorescence image of the single photons traversing the photonic network. (right) All possible output modes of for the three single photons inserted into ports 1, 2 and 4. Experimental results are shown in red and theoretical results are shown in blue. These probabilities are subsequently fit using mathematical models to determine the unitary matrix of the system under study [10].

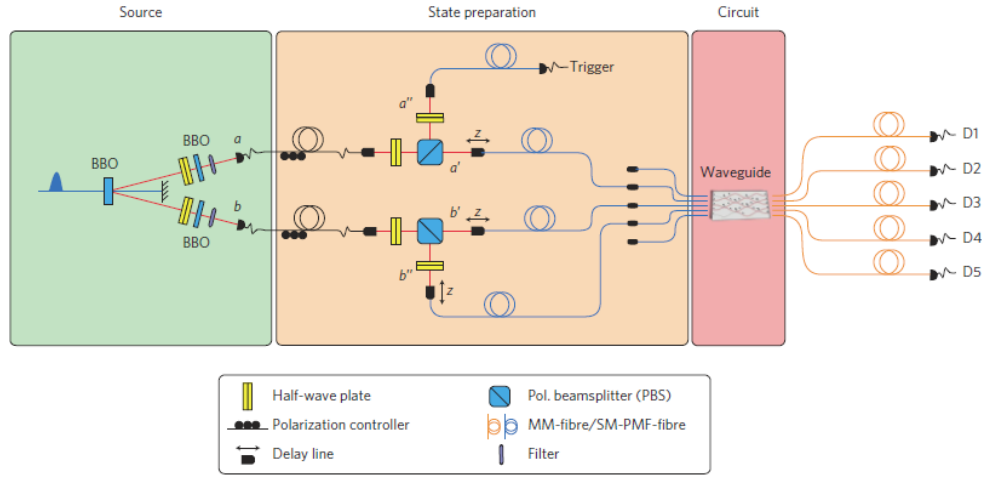


Figure 6 Experimental setup used to pump the photonic system. The first section (green) is the source which down converts a 150fs 395nm pulse into two photons at 790nm through a BBO crystal. Each photon is then subsequently split again by the second BBO crystal, after which the beam is spectrally filtered to ensure the photons are indistinguishable. The light is then coupled into single mode fiber and sent through half wave plate to and beam splitter to split the photons into separate paths. A second beam half-wave plate (in  $a''$  and  $b''$ ) then ensure the polarizations are identical before entering the system. The photons are temporally overlapped using delay lines to observe the Hong-Ou-Mandle dip for two & three photon interference. One of the photons serves as a trigger and the other three can be connected to any input of the chip. The output is detected using avalanche photodiodes [10].

A second group to experimentally demonstrate boson sampling is I. Walmsley et al. In their experiment, an  $M = 6$  input and output (arbitrary 36 element matrix) optical circuit with 10 beamsplitters was analyzed. Before multiphoton injection, the complex valued unitary matrix elements  $u_{ij} = \tau_{ij} e^{i\phi_{ij}}$  were determined by inserting a single photon into port  $i$  and observing at port  $j$ . This probes the amplitude of the matrix element as the probability of detection is  $|\tau_{ij}|^2$ . The relative phase term was measured using two photon interference. The probability that photons inserted at ports  $i_1$  and  $i_2$  and detected at  $j_1$  and  $j_2$  is

$\left| u_{i_1 j_1} u_{i_2 j_2} + u_{i_2 j_1} u_{i_1 j_2} \right|^2$  and was determined using the previously found values for  $\tau_{ij}$ . Recall that the probability of observation for two-photon interference strongly depends upon the phase shift between the two photons. These numbers were then used to predict the output of three and four photon excitation, see Fig. 7 for values, and compared to experimental results, see Fig. 8 [11].

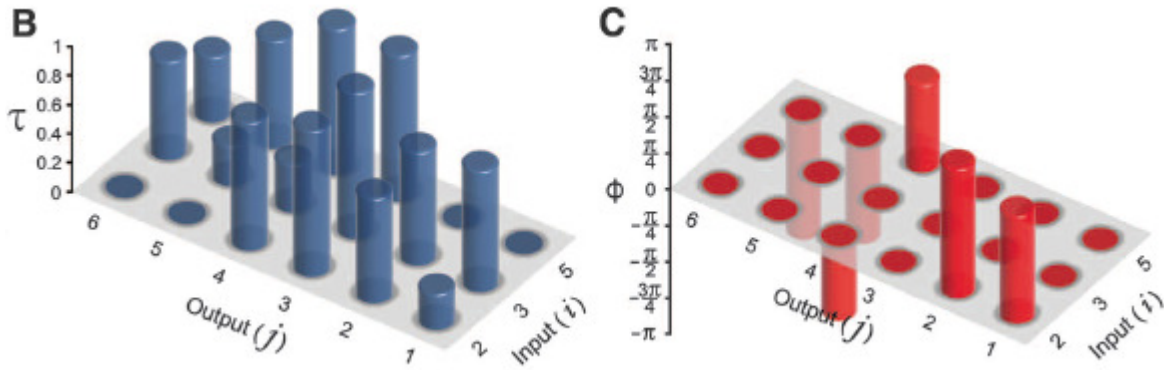


Figure 7 (B) Normalized magnitudes of the unitary matrix elements determined by single photon excitation and observation. (C) Relative phases of the unitary matrix elements determined by two photon excitation. These values were used to predict the three and four-photon excitation response for subsequent experiments [11].

In addition to performing the data analysis, this paper also compared the response of the system to an ideal boson sampling machine. They found that the deviation beyond the expected statistical deviation from the ideal was measureable. Physically, this means that the system wasn't always probing from the statistical distribution of the optical network. This results due to several factors such as partial distinguishability between the incident photons or when the source emits bunched photons instead of single photons.

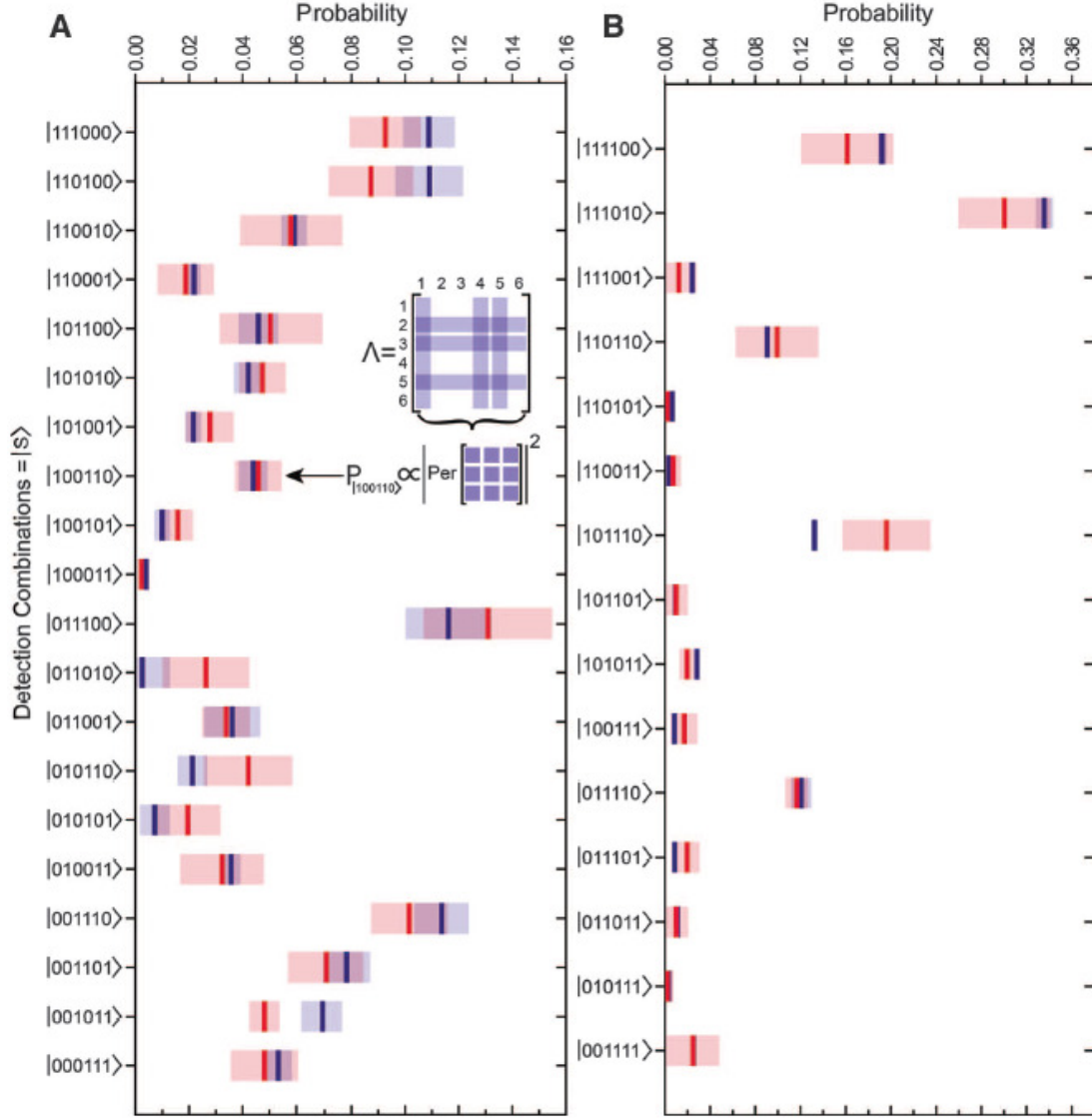


Figure 8 Comparison of the experimental results and predicted results for three photon (A) and four photon (B) excitation. The red is the measured under multi-photon excitation and the blue is the predicted value obtained from individually measuring the elements of the unitary matrix (using single and two-photon excitation). The shaded region represents error in the measurement and theory. The inset in graph (A) illustrates the calculation of the permanent from the unitary matrix  $\Lambda$  for the output mode  $|100110\rangle$  with the input  $|011010\rangle$  [11].

### 3. Outlook for boson sampling

As the study by Walmsley et al pointed out, a significant source of error was due to non-ideally matched photons and a bunching of the photons. These two problems present a significant challenge for the scaling of boson sampling machines towards hundreds or thousands of inputs and outputs. Despite these problems, the current systems have still achieved accurate sampling of the probability distribution, illustrating the resilience of the technique to non-ideal effects. This provides hope to those to see boson sampling as a near-term method to achieve quantum assisted computations.



Additionally, it is believed that the solutions to the problems associated with boson sampling will be easier to solve than realizing a universal quantum computer. However, there is some rebuttal to this thought suggesting that due a large number of bosons will be equally as difficult to maintain coherence as would a large number of qubits. They also question the effort spent to correct such problems for a system that is not universal, as a fully quantum computer (Turing machine) would be. However, as Scott Aaronson, one of the original minds of the boson sampling technique, put it *“BosonSampling would still be of complexity-theoretic interest, for the simple reason that it’s the only natural way known to make permanents show up as a quantum computation’s amplitudes, and the permanent is an extremely interesting and special function in theoretical computer science.”* In essence, boson sampling and the multi-photon interference effects which underlie the process are one of the best and easiest methods for us to probe the quantum world. Fully understanding and modeling its properties could not only help us to achieve quantum assisted computation, but could also help us to better understand the problems related to universal quantum computers and our understanding of the quantum world.

#### 4. Conclusion

Boson sampling offers an unprecedented ability to probe the quantum world and explore a problem that is classically difficult to solve. By inserting a bevy of identical single photons into a photonic network and studying the probabilities of various outputs, the permanents of a unitary matrix can be probed. This method of quantum assisted computation is not fully universal, as an ideal quantum computer would be, but could provide a realistic way to achieve the speed increase promised through quantum computation in the near term. Recently, four independent research groups experimentally verified the hypothesis of Aaronson and Arkhipov, illustrating the use of boson sampling to determine the elements of an arbitrary unitary matrix that would be difficult for a classical computer to compute. With further advances in integrated optical technologies and single photon sources, boson sampling could represent an ability to study and simulate the quantum world at a scale that cannot be achieved except with full quantum computers.

#### References

1. D. Awschalom, L. Bassett, A. Dzurak, E. Hu, and J. Petta, "Quantum Spintronics: Engineering and Manipulating Atom-Like Spins in Semiconductors," *Science* **339**(6124), (2013).
2. A. Stern, and N. Lindner, "Topological Quantum Computation - From Basic Concepts to First Experiments," *Science* **339**(6124), (2013).
3. M. Devoret, and R. Schoelkopf, "Superconducting Circuits for Quantum Information: An Outlook," *Science* **339**(6124), (2013).
4. C. Monroe, and J. Kim, "Scaling the Ion Trap Quantum Processor," *Science* **339**(6124), (2013).
5. T. Wilk, S. Webster, A. Kuhn, and G. Rempe, "Single-Atom Single-Photon Quantum Interface," *Science* **317**(5837), (2007).
6. T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, "Quantum computers," *Nature* **464**(7285), 45-53 (2010).
7. T. Monz, P. Schindler, J. Barreiro, M. Chwalla, D. Nigg, W. Coish, M. Harlander, W. Hansel, M. Hennrich, and R. Blatt, "14-Qubit Entanglement: Creation and Coherence," *Phys. Rev. Lett.* **106**(13), (2011).
8. G. Waldherr, Y. Wang, S. Zaiser, M. Jamali, T. Schulte-Herbruggen, H. Abe, T. Oshima, J. Isoya, J. F. Du, P. Neumann, and J. Wrachtrup, "Quantum error correction in a solid-state hybrid spin register," *Nature* (2014).

9. S. Aaronson, and A. Arkhipov, "The Computational Complexity of Linear Optics," in *Proceedings of the 43rd Annual ACM Symposium on Theory of Computing*, (New York, NY, 2011), pp. 333-342.
10. M. Tillmann, B. Dakic, R. Heilmann, S. Nolte, A. Szameit, and P. Walther, "Experimental Boson Sampling," *Nature Photon.* **7**540 (2013).
11. J. B. Spring, B. J. Metcalf, P. C. Humphreys, W. S. Kolthammer, X. M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, D. Kundys, J. C. Gates, B. J. Smith, P. G. R. Smith, and I. A. Walmsley, "Boson Sampling on a Photonic Chip," *Science* **339**(6121), 798-801 (2012).
12. C. Gerry, and P. Knight, *Introductory Quantum Optics* (Cambridge University Press, Cambridge, UK, 2008).
13. J. S. Fokonas, H. Lee, Y. A. Kelaita, and H. Atwater, "Two-Plasmon Quantum Interference," *Nature Photon.* **8**(4), 317 (2014).
14. A. Crespi, R. Osellame, R. Ramponi, D. Brod, E. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, and F. Sciarrino, "Integrated multimode interferometers with arbitrary designs for photonic boson sampling," *Nature Photon.* **7**(7), (2013).
15. M. Broome, A. Fedrizzi, S. Rahimi-Keshari, J. Dove, S. Aaronson, R. Ralph, and A. White, "Photonic Boson Sampling in a Tunable Circuit," *Science* **339**(6121), 794-798 (2013).