Ian Stevenson

The Pound Drever Hall (PDH) stabilization technique is the subject of [1], in fact it derives its name from its two primary authors Drever and Hall (Pound originally applied the technique to masers decades prior). In this context stabilization refers to locking the central frequency of a laser to some know resonance and reducing the fast phase fluctuations of said laser. In the literature these lasers are referred to as narrow band, though all lasers are quite narrow band with full width half maximum (FWHM) frequencies (often in the range of a few MHz for diode lasers for example) just a small fraction of their central frequency (often in the range of 10^14 Hz), these lasers are several orders of magnitudes narrower and lasers with sub-hertz line widths have even been reported [2]. These narrow band lasers often find applications in extreme situations such as the LIGO project [3], in the spectroscopy of states with line width's on the order of a hertz [2], or in atomic clocks where precision is paramount.

The PDH technique is not the only way to achieve narrow band lasers, but it is the best. Side locking is one common example that predates PDH and is discussed briefly in [1]. The idea is to use the side of a Fabry-Perot transmission fringe as the frequency discriminator and shown in figure 1. If

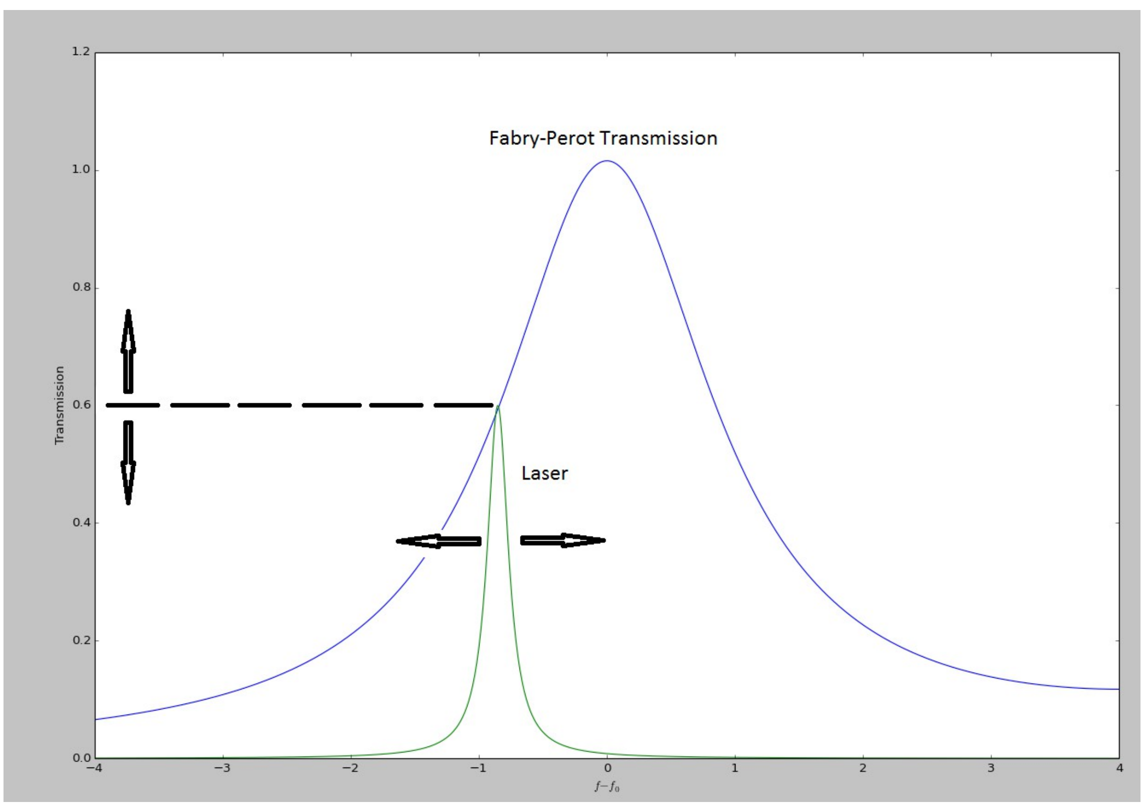

Figure 1. Side Locking, turning frequency changes into changes in observed voltage level

the laser frequency is too high then the feedback network will produce an error signal opposite in sign to if the laser frequency is too low. As discussed in [1] the value in this technique is its simplicity but it isn't without its problems. First it is desirable for the feedback network to have a large bandwidth but that fails for this technique as the response of the Fabry-Perot fringe is only linear with respect to frequency in a small range. Second, this technique mixes intensity noise with frequency noise and though there are ways to reduce this problem they result in short lock lifetimes. There are other more specialized techniques for laser stabilization, for example, direct optical feedback can be applied to diode lasers to reduce their line width [5], but this suffers from requiring low finesse cavities which PDH stabilization does not. In summary PDH is the premier technique for laser stabilization.

The PDH stabilization technique works via three basic pieces. First a laser is phase modulated. Second the laser is sent into a high finesse Fabry-Perot cavity and the sum of the reflection/light coupled back out of the cavity is measured. Third this error signal is demodulated and amplified before being sent back to the laser controller. This process is illustrated beautifully graphically in figure 2, which is borrowed from [1]. On a conceptual level the PDH locking scheme works as both a phase detector and a frequency detector depending on the time scale of the deviation from ideal of the laser and this idea is illustrated in figure 2. When the fluctuations are on a time scale smaller than the response time of the cavity it is a phase detector. In this regime the cavity acts as a fly wheel of sorts. The light coupled back out of the cavity added to the reflection that is now out of phase becomes the error signal. When the fluctuations are on a longer time scale than the response time of the cavity the lock acts as a frequency detector. Here one of the sidebands will be closer to resonance (depending on

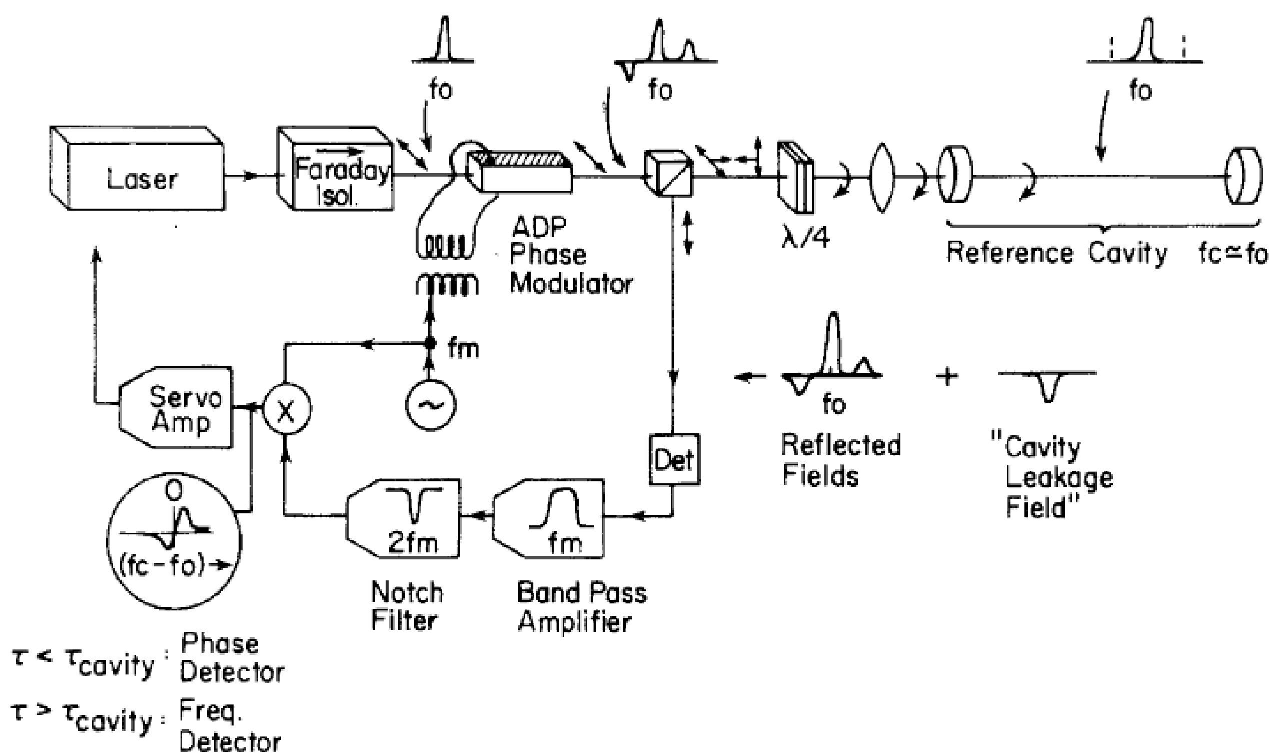

Figure 2. Drever-Hall's Block Diagram Explaining Their Locking Scheme

whether the laser drifts high or low in frequency) and this becomes the error signal that drives the laser back to the center frequency of the cavity resonance. Both of these modes are desirable in a locking scheme; the phase detection mode actually reduces the line width of the laser by eliminating the fast frequency excursions and the frequency detection mode the lock literally locks the laser to the center of the cavity and is obviously quite useful in any sort of application that requires the laser to be at a known frequency.

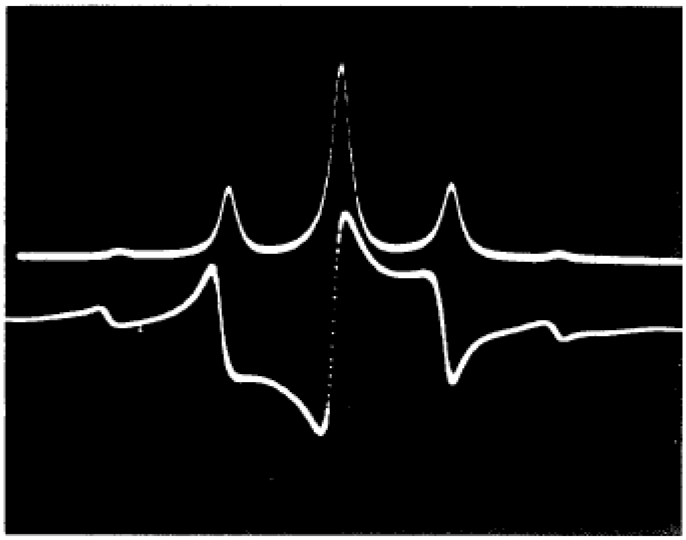

Figure 3. Top trace: signal detected by photo detector, Bottom trace: PDH error signal

The electronics depicted in figure 2, from the detector in the block diagram thru to the servo amp, function to turn the initially observed signal, which is the top trace in figure 3, into the error signal, the bottom trace in figure 3. After the photo detector the signal is run thru an bandpass filter and a phase shifting filter to get a 90 degree phase difference between this signal and the local oscillator. Then the signal is demodulated to move the error information from the local oscillator frequency down to DC and finally it is run thru the servo amp to bring it up to a reasonable signal level.

In their paper Drever and Hall [1] indirectly highlighted the technical difficultly in evaluating the performance of a laser lock. To evaluate the precision of a laser lock one needs a more accurate frequency reference than the laser itself. On common method for achieving this is to beat two identical lasers both locked to an external Fabry-Perot and measure this signal on a photo diode. The frequency width of the beat signal then represents the noise on the lasers and is a way to measure the laser line width. In their original experiment Drever and Hall locked two different lasers to the same Fabry-Perot and estimated the line width of the lasers was 87 Hz.

A brief theoretical treatment of PDH locking is as follows, credit to [3]. The laser field, post

modulation is approximately: $E = E_0\, e^{i\,\omega\,t + B\sin(\Omega\,t)}$ and this can be expanded to a first order approximation to a sum of Bessel functions: $E_i = E_0\left[J_0(B)\, e^{i\,\omega\,t} + J_1(B)\, e^{i\,(\omega+\Omega)\,t} - J_1(B)\, e^{i\,(\omega-\Omega)\,t}\right]$.

Once this field is reflected from the cavity it has the form $E_r = \Gamma(\omega)\, E_i$ with $\Gamma(\omega) = \frac{r\left(e^{i\,\phi} - 1\right)}{1 - r^2\, e^{i\,\phi}}$ and $\phi = \frac{\omega}{\Delta\, v_{fsr}}$ is the phase build up in the cavity. Once this field is measured by a photo detector it will be the power, not the field observed as current, thus $P_r = |E_r|^2$, expanding this gets messy, but is necessary:

$$P_r = P_c\,|\Gamma(\omega)|^2 + P_s\left[|\Gamma(\omega+\Omega)|^2 + |\Gamma(\omega-\Omega)|^2\right] + 2\sqrt{P_c\,P_s}\,\left[\Re\left(\Gamma(\omega)\,\overline{\Gamma(\omega+\Omega)} - \overline{\Gamma(\omega)}\,\Gamma(\omega - \Omega)\right)\cos(\Omega\,t) + \Im\left(\left(\Gamma(\omega)\,\Gamma(\overline{\omega+\Omega}) - \Gamma(\overline{\omega})\,\Gamma(\omega - \Omega)\right)\sin(\Omega\,t)\right)\right] + O(2\,\Omega)$$

where Pc is the power in the carrier and Ps is the power in the sidebands this waveform is seen in the top trace of figure 3. Our demodulation stage picks off the sine term so the error signal is $\varepsilon = 2\sqrt{P_c\,P_s}\,\Im\left(\Gamma(\omega)\,\Gamma(\overline{\omega+\Omega}) - \Gamma(\overline{\omega})\,\Gamma(\omega-\Omega)\right)$. When plotted this give the characteristic PDH error signal seem in the bottom trace in figure 3. When the laser field is nearly in resonance, things change slightly and it is easy to consider what happens if the cavity length changes slightly (a change in the laser wavelength could be viewed as a relative change in the length of the cavity). First the sidebands are totally reflected so $\Gamma(\omega \pm \Omega) = -1$ this leads to $P_r = 2\,P_s - 4\sqrt{P_c\,P_s}\,\Im(\Gamma(\omega))\,\sin(\Omega\,t)$. Here our phase is $\phi = 2\,\pi\,N + \frac{4\,\pi\,\delta\,L}{\lambda}$ and writing the reflection coefficient in terms of the phase leads to: $\Gamma(\delta\,L) = \frac{4\,i\,F\,\delta\,L}{\lambda}$ where F is the cavity finesse. Substituting into the power reflected: $P_r = 2\,P_s - \frac{16\sqrt{P_c\,P_s}\,F\,\delta\,L\,\sin(\Omega\,t)}{\lambda}$ and since the mixer picks off the sine component of this signal the error signal becomes: $\varepsilon = \frac{16\sqrt{P_c\,P_s}\,F\,\delta\,L}{\lambda}$. This leads to a linear relation between frequency excursion and error signal (which is preferred) and is what provides the lock.

References:

[1] Drever, R. W. P. (1983). "Laser phase and frequency stabilization using an optical resonator". *Appl Phys B* **31** (2): 97. doi:10.1007/BF00702605

[2] Alnis, J., et al. "Subhertz linewidth diode lasers by stabilization to vibrationally and thermally compensated ultralow-expansion glass Fabry-Pérot cavities." *Physical Review A* 77.5 (2008): 053809.

[3] Black, Eric. "Notes on the Pound-Drever-Hall technique". *LIGO Technical Note*. Retrieved 9 April 2014