TALBOT EFFECT

PHYS 522 TERM PAPER

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ABSTRACT

The purpose of this paper is to provide a short review to an interesting observation/effect in Physics known as the Talbot effect - a phenomenon in which patterns are repeated after a periodic distance known as the Talbot length. This paper starts out by introducing the Talbot effect and different areas where this effect has been observed. After that, a theoretical analysis of the classical self-imaging effect (another name for Talbot effect) is discussed. The bulk of this paper focuses on observation of the Talbot effect in the temporal domain using two-photon frequency combs and then it concludes with a summary in addition to potential applications of the self-imaging effect using two-photon frequency combs.

1. INTRODUCTION

In 1836, Henry Fox Talbot discovered that a periodic structure produced images of itself at certain regular intervals [1]. Talbot noticed that when he illuminated a Fraunhofer diffraction grating with a very small white light source, he observed a periodic repetition of color bands that looked like the thin slits of the grating [1]. Then in 1881, Lord Rayleigh showed that this effect was a result of diffraction interference of plane waves by gratings [1]. The periodic distance between these color bands is known as the Talbot distance and it is equal to d^2/λ, while the primary Talbot distance is $2d^2/\lambda$.

Around the middle of the 20th century, Cowley and Moodey extensively investigated the Fourier images produced from periodic objects that could act as their own imaging system [1, 2, 3, 4, 5]. They developed a theory based on Fresnel diffraction equations in which they found that the periodic structure was replicated at multiples of a longitudinal distance zT. The term 'self-imaging' was introduced by Montgomery in 1966 and is now used alongside with the 'Talbot effect' to refer to the same phenomenon [1, 6].

Space-Time duality properties like paraxial diffraction of optical beams in space and dispersion of narrowband pulses in time has also been explored and has led to the observation of the Talbot effect in periodic temporal signals propagating through a dispersive medium, first described by Jannson and Jannson in 1980 [1, 7].

Other observations of the Talbot effect have been shown using surface plasmons, multimode interference effects and non-linear optics, while there have also been proposals for observing this effect using metamaterials [1]. However, this paper will emphasize on an observation in quantum optics. Section 2 provides a theoretical discussion of the classical self-imaging effect and then Section 3 focuses on a demonstration of Talbot Effect in quantum optics - coherence revivals using two-photon frequency combs (which will occasionally be referred to as quantum Talbot effect in the temporal domain). The paper ends with a conclusion and possible applications of coherence revivals using two-photon frequency combs.

2. CLASSICAL SELF-IMAGING EFFECT

Theory

The diffraction grating is a periodic structure (periodic in the x-direction) and can be represented as a Fourier series given below:

$$t(x) = \sum_m a_m e^{\frac{i2\pi mx}{d}},$$

where a_m is the Fourier coefficient and d is the grating period.

If the grating is normally illuminated by a plane wave $U = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$, then the beam is diffracted by an angle given by:

$$\sin(\theta_d) = \frac{m\lambda}{d}$$

Assuming the k-vector is only in the z-direction, then

$$k_z = kcos(\theta_d) = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{m\lambda}{d}\right)^2} \approx \frac{2\pi}{\lambda}\left[1-\frac{1}{2}\left(\frac{m\lambda}{d}\right)^2\right]$$

After propagating a distance z, the field is given by

$$U(x,z) = \sum_m a_m e^{\frac{i2\pi mx}{d}} e^{ik_z z}$$

$$= e^{\frac{i2\pi z}{\lambda}} \sum_m a_m e^{\frac{i2\pi mx}{d}} e^{-i2\pi m^2 \frac{\lambda}{2d^2} z}$$

It can be seen that the summation term will repeat itself whenever $z\lambda/2d^2 = p$ $(p\ is\ an\ integer)$. This will result in $z = 2p\frac{d^2}{\lambda}, p = 1, 2, 3, \ldots$ - This is the primary Talbot distance

Importantly, if we divide z in the previous line by 2, the summation term will still repeat itself but there will be an extra $e^{i\pi}$ term which corresponds to a spatial shift in z. Thus, the Talbot distance is $z_T = z/2$, while the primary Talbot distance is $2z_T$. The primary and secondary Talbot distances along with other patterns are illustrated in Fig. 1.

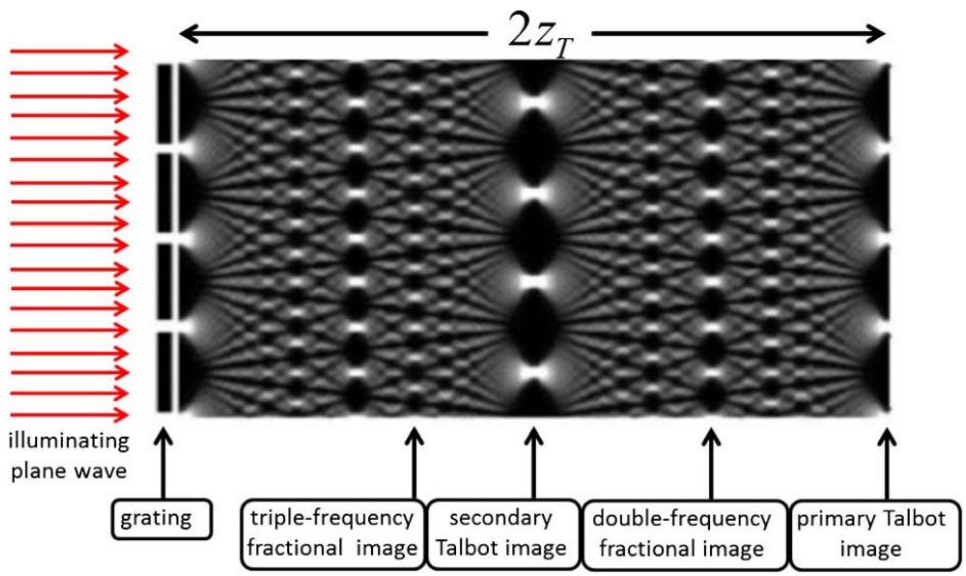

Fig. 1. The Talbot effect for a monochromatic light, shown as a "Talbot carpet." (Image from [1]). The grating image is shifted by half a period in the secondary Talbot image while it is perfectly aligned in the primary Talbot image. Also, at a quarter of the primary Talbot image, the grating image is doubled (image period is half of grating period).

3. TALBOT EFFECT IN QUANTUM OPTICS

The first theoretical proposal to explore the Talbot effect in quantum optics was put forward by Luo et al. where they considered quantum ghost imaging with entangled photons and quantum lithography [1, 8]. There were also some other studies by various research groups but this section will focus on the self-imaging effect of entangled photon pairs in the time domain, first proposed by Victor Torres-Company et al. in 2009 [1, 9].

In the analysis by Torres-Company and colleagues, they showed that the temporal self-imaging effect (coherence revivals) can exist for the two-photon probability (second-order correlation function) of two-photon frequency combs [9]. The coherence revivals of the two-photon frequency combs are observed at periodically spaced dispersion values.

3.1. **Theory** *(adapted from paper by Torres-Company et al. [9])*

The general scheme is to consider a two-photon wave packet where the signal and idler propagate through two different dispersive media and then the arrival times of the photons are detected and correlated. The probability of detecting an idler photon at a time t_1 given the signal photon arrives at time t_2 is known as the second-order correlation function and is given by:

$$G^{(2)}(t_1, t_2) \equiv \langle\Psi|E_s^-(t_1)E_i^-(t_2)E_i^+(t_2)E_s^+(t_1)|\Psi\rangle$$

The delay between the signal and idler photon is $\tau = t_2 - t_1$.

In the two-photon case,

$$G^{(2)}(\tau) = |\psi(\tau)|^2$$

where $\psi(\tau)$ is considered as the two-photon probability amplitude and can be represented as

$$\psi(\tau) = \int d\omega \Phi(\omega) e^{\frac{i\Phi_{2eff}\omega^2}{2}} e^{-i\omega\tau}$$

$\Phi(\omega)$is the two-photon spectrum and $\Phi_{2eff} = \Phi_{2s} + \Phi_{2i}$ is the joint group delay dispersion parameter of the dispersive media in the arms of the signal and idler photons.

If the two-photon spectrum is a frequency comb with $\Phi(\omega) = \phi(\omega)\sum_{n=-\infty}^{\infty}\delta(\omega - n\Delta\omega)$, then the probability amplitude repeats itself whenever $\frac{\Phi_{2eff}\Delta\omega^2}{2} = 2N\pi \leftrightarrow \Phi_{2eff} = \frac{NT^2}{\pi}$.

Actually, it is important to note that coherence revivals will occur whenever $\Phi_{2eff} = \frac{NT^2}{L\pi}$ where N and L are any coprime integer numbers.

3.2. **Experimental Demonstration**

In a recent article by Joseph Lukens and colleagues on the generation of two-photon correlation trains, the first experimental demonstration of this temporal self-imaging of entangled photons was reported [10]. Their experimental setup is shown in Fig. 2.

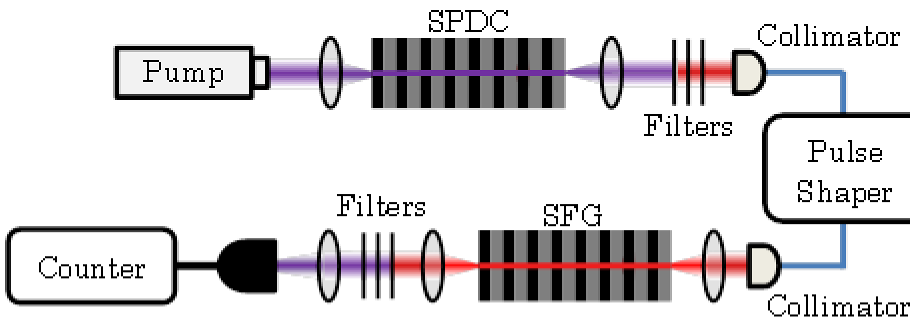

FIG. 2 Experimental Setup to observe temporal self-imaging using entangled photons (Image from [10]).

In their experiment, they pumped a periodically poled lithium niobate (PPLN) waveguide with a continuous-wave pump beam at ~774 nm to generate frequency-entangled photons at 1548nm through spontaneous parametric down conversion (SPDC). A set of filters were placed after the SPDC portion to remove unwanted frequency components such as the pump beam. The created entangled photons were coupled to an optical fiber through a collimator and then quadratic dispersion (Φ_{2eff}) was applied using a commercial pulse shaper. Another collimator was used to couple the entangled photons from the pulse shaper into another waveguide (phase-matched with the first waveguide) where they recombined through sum frequency generation (SFG). Again, another set of filters was used to remove photons that did not recombine. Finally, the SFG photons were detected using an avalanche photodiode and the two-photon correlation function was measured.

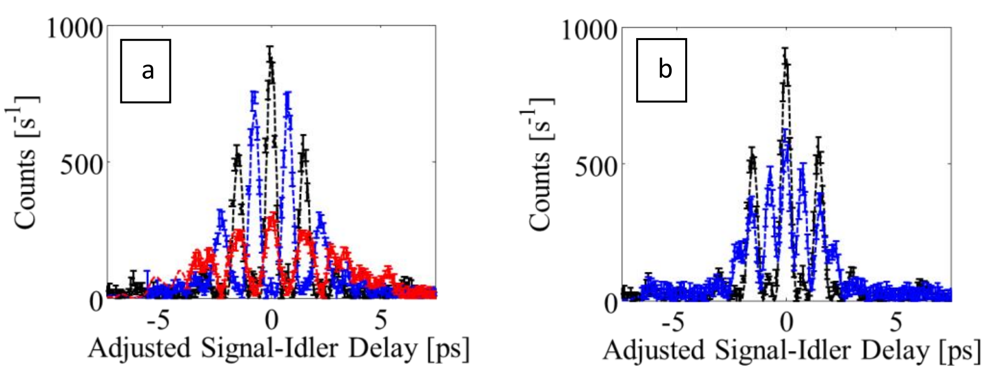

Fig. 3. Two-photon correlation functions. (a) Black curve represents $\Phi_{2eff} = 0$, Blue curve for $\Phi_{2eff} = T^2/2\pi$ (half Talbot), Red curve for $\Phi_{2eff} = T^2/\pi$ (full Talbot), (b) Black curve for $\Phi_{2eff} = 0$ and Blue curve for $\Phi_{2eff} = T^2/4\pi$ (quarter Talbot) (Image from [10]).

From their results, it can be clearly seen that they obtained similar waveform patterns when the dispersion was set to 0 and T^2/π $(N = 1)$. Then when $N = 1/2$ (half Talbot), the waveform pattern is shifted by half of its period, while when $N = 1/4$ (quarter Talbot), the waveform doubled. The authors also pointed out that the damping in the correlation function was as a result of finite linewidth (the features would have been enhanced with a finer linewidth).

4. CONCLUSION

Short and concise, this paper has provided a brief insight into the self-imaging effect that was discovered almost 200 years ago. The discussion provided a historical background of the Talbot effect and then tied it to self-imaging with entangled photon pairs – the first demonstration of coherence revivals in two-photon frequency combs was included. A potential application of the quantum Talbot effect in the temporal domain is repetition-rate multiplication of two-photon pulse trains [10] – as was seen in the quarter Talbot example in Fig. 3b. Another application is the transmission of frequency standards through optical fibers while bypassing the need for dispersion cancellation [9].

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