Laser Cooling

ECE 695: Quantum Optics

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The thermal energy per particle at a temperature T can be equated to kinetic energy:

$$E = \frac{1}{2} k_B T = \frac{1}{2} m v_x^2$$

$$v_x^{rms} = \sqrt{\frac{k_B T}{m}}$$

In Fig 11.1, an atom is traveling in the direction opposite to the propagation direction of light. Due to the Doppler effect, the frequency of the light seen by the atom is shifted and this shift depends on the velocity of the atom. In this configuration, if the laser is slightly red-detuned from resonance by δ, and the atom has an atomic transition frequency ν_0, the Doppler-shifted frequency can be calculated:

$$\nu_L' = \nu_L \left(1 + \frac{v_x}{c}\right) = (\nu_0 + \delta)\left(1 + \frac{v_x}{c}\right) \cong \nu_0 + \delta + \frac{v_x}{c}\nu_0$$

By setting the detuning of the laser $\delta = -\frac{v_x}{c}\nu_0$, the atom would experience resonance if it is travelling only in the x-direction.

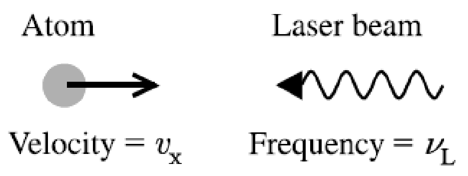


Fig. 11.1 In Doppler cooling, the laser frequency is tuned below the atomic resonance by δ. The frequency seen by an atom moving towards the laser is Doppler-shifted up by $\nu_0(v_x/c)$.

Once the atom absorbs a photon, it spontaneously emits it in a random direction. After a cycle of absorbing and releasing a photon, the momentum of the atom would change by:

$$\Delta p_x = -\frac{h}{\lambda}$$

This arises from the fact that the photon has a momentum of $\hbar k$ and that absorption causes the change of $\hbar k$ in the negative x-direction while spontaneous emission causes net average zero

momentum change. The net momentum change at each cycle implies that there is force applied to the atom in the process of absorption and emission:

$$F = \frac{dp_x}{dt} \cong \frac{\Delta p_x}{2\tau} = -\frac{h}{2\lambda\tau}$$

This force, acting in the negative x-direction, causes deceleration of the atom:

$$F = -\frac{h}{2\lambda\tau} = ma \rightarrow a = -\frac{h}{2m\lambda\tau}$$

If the atom emits a photon in the same direction as the incident photon (stimulated emission), this process cancels the net momentum change and thus does not contribute to the atomic deceleration. The number of cycles and the minimum time required to slow down the atom to the minimum velocity are given by:

$$N_{stop} = \frac{mu_x}{|\Delta p_x|} = \frac{mu_x\lambda}{h}, \qquad t \cong N_{stop} \times 2\tau = \frac{2mu_x\lambda\tau}{h}$$

As the Doppler cooling process continues, the required detuning δ changes due to the fact that δ depends on the velocity of the atom. The cooling process becomes ineffective when δ becomes comparable to the natural width (Δν) of the transition. From this fact, the minimum temperature of the atom can be approximated:

$$k_B T_{min} \sim h\Delta\nu \rightarrow T_{min} \sim \frac{\hbar}{k_B\tau}$$

If one considers the net rate (R) of absorption and emission, here I_s is the saturation intensity and γ is the natural decay rate of the transition, the force experienced by the atom becomes:

$$F_x = -\hbar k \times R(I,\Delta), \qquad R(I,\Delta) = \frac{\gamma}{2}\left(\frac{\frac{I}{I_s}}{1 + \frac{I}{I_s} + \left[\frac{2(\Delta + kv_x)}{\gamma}\right]^2}\right)$$

When the atom has slowed down to a low speed, the laser will start to accelerate the atom again in the opposite direction. In order to counter this undesired effect, an additional laser needs to provide a photon in the opposite direction.

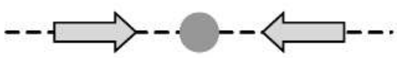


Fig. 11.4 Two counter-propagating lasers are used to produce the optical molasses cooling effect.

If the hot atom is moving in the positive x-direction, the lasers should be set up such that:

$$F = F_{+} + F_{-}, \qquad F_{-} \gg F_{+}$$

In the low-temperature limit with $|kv_x| \ll \Delta, \gamma$ the net force can be calculated:

$$F_x(I,\Delta) = -\alpha v_x = \frac{8\hbar k^2 \Delta}{\gamma}\left(\frac{\frac{I}{I_s}}{1+\frac{I}{I_s}+\left[\frac{2(\Delta + kv_x)}{\gamma}\right]^2}\right)v_x$$

Here α is the damping coefficient. When the detuning is negative for the counter-propagating cooling beams, the damping coefficient becomes positive. In this configuration, the atoms are like optical molasses. Although the maximum damping force is achieved with $\Delta = -\frac{\gamma}{\sqrt{12}}$, the lowest temperature can be achieved at a different frequency. By balancing the cooling and heating rates:

$$\left(\frac{dE}{dt}\right)_{cooling} = F_x v_x = -\alpha v_x^2$$

$$\left(\frac{dE}{dt}\right)_{heating} = \frac{D_p}{m}$$

$$\left(\frac{dE}{dt}\right)_{cooling} + \left(\frac{dE}{dt}\right)_{heating} = -\alpha v_x^2 + \frac{D_p}{m} = 0$$

Thus,

$$\alpha v_x^2 = \frac{D_p}{m} \rightarrow T = \frac{D_p}{\alpha k_B}$$

The momentum diffusion coefficient $\frac{D_p}{m}$ is given by:

$$D_p = \frac{1}{2}\frac{d\langle p_x^2\rangle}{dt}$$

At the cold temperature limit, this coefficient becomes

$$D_p = \hbar^2 k^2 \gamma\left(\frac{\frac{I}{I_s}}{1+\frac{I}{I_s}+\left[\frac{2(\Delta + kv_x)}{\gamma}\right]^2}\right)$$

And then the minimum temperature can be calculated:

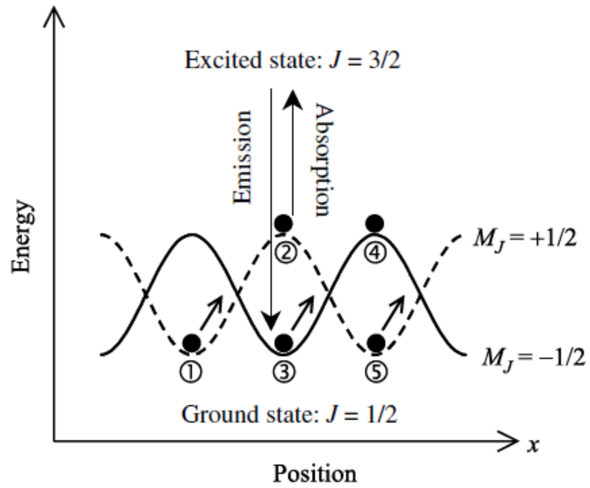
$$T = \frac{D_p}{\alpha k_B} = -\frac{\hbar\gamma}{8k_B}\frac{\left(1+\frac{I}{I_s}+\frac{4\Delta^2}{\gamma^2}\right)}{\Delta/\gamma}$$

Assuming the laser intensity is far below the saturation intensity, and $\Delta = -\frac{\gamma}{2}$, this temperature becomes:

$$T_{min} = \frac{\hbar\gamma}{2k_B} = \frac{\hbar}{2k_B\tau}$$

This minimum temperature is known as the Doppler limit.

However, the experimental results of the Doppler cooling show the interference of the counter propagating beams lead to a mechanism called Sisyphus cooling:



Because of the interference of the electric fields of the counter propagating beams, the potential experienced by the moving atom becomes periodic. The AC Stark effect induces different energy splitting on the atoms' ground levels as they travel along the lasers. With careful manipulation of the lasers, the atoms can follow the path (1) -> (2) -> (3) -> (4) -> (5) which causes the atoms to lose energy. The minimum temperature that the atoms can reach through the Sisyphus mechanism is called the recoil limit.

$$\frac{1}{2}k_B T_{recoil} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m} \rightarrow T_{recoil} = \frac{h^2}{mk_B\lambda^2}$$

In order to trap free atoms, it is necessary to implement 3 sets of counter propagating lasers with a magnetic quadrupole field as shown in Figure 11.6. The magnetic quadrupole field is provided by a pair of coils with the currents of the same magnitude flowing in the opposite directions. The magnetic field at the center of the trap becomes zero. This gives rise to attractive force for the states $M_J > 0$ and repulsive force for the states $M_J < 0$.

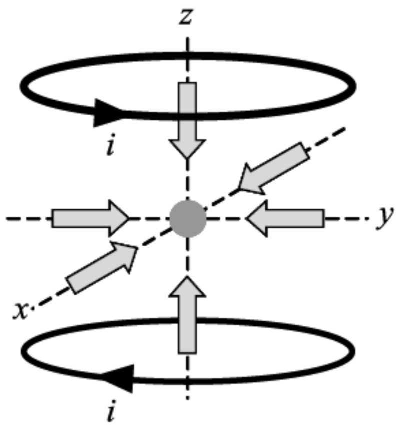
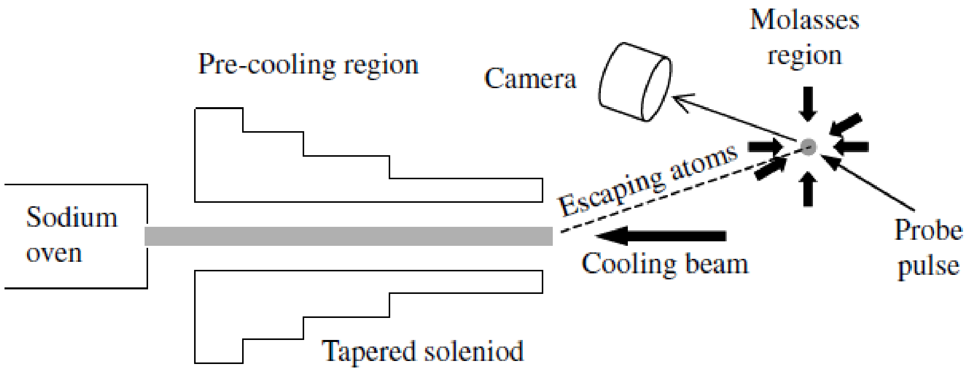


Fig. 11.6 The magneto-optic trap.

While used with the three pairs of beams for the optical molasses effect, the coils therefore help compress atoms in a greater number.



The tapered solenoid in the pre-cooling region shown above cools down atoms to about 2.5K. When the atoms leave the apparatus, they get captured by the magneto-optical traps (MOT) with three pairs of counter-propagating beams. Here the atoms get cooled down to a milli-Kelvin scale. The atoms trapped in the MOT get detected by imaging pulses which can determine the velocity of the atoms.

In statistical mechanics, a fraction of atoms accumulated in the zero velocity states is known as Bose-Einstein condensate. The transition temperature at which condensation occurs is given by:

$$T_C = 0.0839 \frac{h^2}{mk_B}\left(\frac{N}{V}\right)^{2/3}$$

Here N is the number of particles in volume V and m is the atomic mass. Assuming that the particles do not interact with one another and that the potential energy of these particles is zero, we can define the lowest energy state as E = 0. The density of states for bosons is given by:

$$g(E)dE = 2\pi(2S+1)\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}dE$$

This result can be derived from 3D density of states with multiple spin states:

$$g^{3D}(k)dk = \frac{4\pi k^2 dk}{\left(\frac{2\pi}{L}\right)^3} = V\frac{k^2}{2\pi^2}dk$$

$$g(k) \equiv \frac{g^{3D}(k)}{V} = \frac{k^2}{2\pi^2}$$

$$g(\omega)d\omega = 2 \times g(k)dk$$

Here the factor 2 is to account for the two polarizations.

$$g(\omega) = \frac{2g(k)}{d\omega/dk}$$

$$= \frac{\omega^2}{\pi^2 c^3}$$

Or,

$$g(E) = 2 \times \frac{g(k)}{dE/dk}$$

Therefore,

$$g(E)dE = \frac{1}{2\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2} E^{1/2}dE$$

The spin multiplicity becomes (S+1) for non-interacting particles. So we get:

$$g(E)dE = \frac{(2S+1)}{4\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2} E^{1/2}dE$$

By integrating all energy levels across all possible states, we get the number/density of atoms:

$$\frac{N}{V} = \int_0^\infty n_{BE}(E)g(E)dE$$

Here, $n_{BE}(E)$ follows the Boson statistics. Assuming the spin number is 0, the spin multiplicity becomes unity.

$$\frac{N}{V} = 2\pi\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}}{\exp\left[\frac{E-\mu}{k_BT}\right]-1}\,dE = N_0(T) + 2\pi\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}}{\exp\left[\frac{E-\mu}{k_BT}\right]-1}\,dE$$

The term $N_0(T)$ is inserted to account for the case when the mean energy of state μ approaches zero and E = 0. $N_0(T)$ can be calculated using Taylor expansion:

$$N_0(T)V = \frac{1}{\left(1-\left(\frac{\mu}{k_BT}\right)+\cdots\right)-1} = -\frac{k_BT}{\mu}$$

The expression above shows how close the mean energy μ approaches to zero with a given number of atoms in the system. The condensation becomes impossible when atoms start taking up energy levels where E>0. Therefore,

$$\frac{N}{V} = 2\pi\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}}{\exp\left[\frac{E}{k_BT}\right]-1}\,dE = 2\pi\left(\frac{2mk_BT_c}{h^2}\right)\times 2.315$$

From this relation, we obtain:

$$T_c = 0.0839\,\frac{h^2}{mk_B}\left(\frac{N}{V}\right)^{2/3}$$

System	Boson	T_c
Liquid He	He 4	2.17 K
Dilute atom gases	Rb 87, Na 23, Li 7, H 1, etc.	$\sim 10^{-7}K$
Superconductors	Electron Cooper pairs	Up to ~100 K
Semiconductor	Exciton	~ 10K
Neutron star	Neutron Cooper pairs	$\sim 10^{9}K$