Light-matter interaction in cavities and other photonic structures

Lecture 20

Phys 552/ECE 695 Quantum Optics and Photonics

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The lecture is mostly based on the material from the book "Quantum Optics: An Introduction" by Mark Fox (FQ) [1]

1. Spontaneous emission in free space

The transition rate W for spontaneous emission of simple two-level system (atom) can be described by Fermi's golden rule:

$$W = \frac{2\pi}{\hbar^2}\left|M_{12}\right|^2 g(\omega), \tag{1}$$

where M_{12} is a matrix element of external field-matter interaction, $g(\omega)$ is density of the field at the transition frequency, hereafter referred as photonic density of states. Since there is no applied external field, our atomic system interacts with vacuum modes:

$$\left|M_{12}\right|^2 = \xi^2\mu_{12}^2 E_{vac}^2 = \xi^2\frac{\mu_{12}^2\hbar\omega}{2\varepsilon_0 V}, \tag{2}$$

where ξ - normalized dipole orientation (for the averaged orientation ξ=1/3).

Photonic density of states (in phase space) can be calculated by standard approach as

$$g(\omega) = \frac{\omega^2 V}{\pi^2 c^3}. \tag{3}$$

Combining together formulas (1), (2), (3):

$$W = \frac{\mu_{12}^2\omega^3}{3\pi\varepsilon_0\hbar c^3}. \tag{4}$$

It should be noted that the same result can be obtained if considering single oscillating dipole. The well-known Larmor formula expressing dipole-emission power is $P_0 = \frac{\mu^2\omega^4}{12\pi\varepsilon_0 c^3}$.

The spontaneous emission rate can be also related with Einstein coefficients:

$$W = \tau_R^{-1} = A_{21} = \frac{\hbar\omega^3}{\pi^2 c^3}. \tag{5}$$

2. Optical cavity

Here we would like to summarize planar cavity properties and the key parameters describing it.

Below we list typical cavity parameters:

- resonant mode wavelength/frequency: $\lambda_m = \frac{2nL_{cav}}{m}$, $f_m = m\frac{c}{2nL_{cav}}$;
- spectral width: δf;
- round trip phase shift: $\varphi = 2kL_{cav} = \frac{4\pi nL_{cav}}{\lambda}$;
- free spectral range (FSR): $\Delta f = f_m - f_{m-1} = \frac{c}{2nL_{cav}}$;

- finesse (resolving power): $F = \Delta f \,/\, \delta f$;
- quality factor: [= 2π (energy stored in the system at resonance)/(energy lost in a cycle of oscillation)], $Q = \dfrac{f}{\delta f} = \dfrac{\omega}{\delta\omega} = \omega\tau = mF$;
- survival factor:[=portion of photons lost in one round trip], $S = R_1 R_2$ (for the lossless planar cavity);
- photon decay time: $\tau = \dfrac{t_{RT}}{1-S} = \dfrac{1}{\delta\omega}$
- photon decay rate: $\kappa = \dfrac{1}{\tau} = \delta\omega$

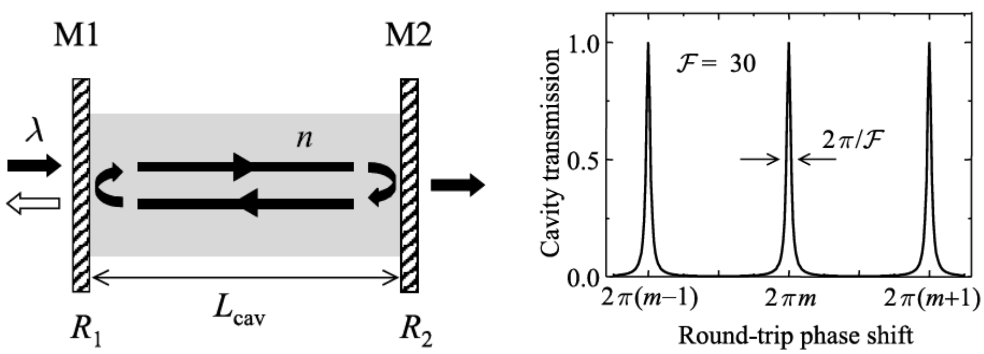

Fig. 1. Left panel: schematic of the planar cavity with two parallel mirrors M1 and M2 of reflectivity R1 and R2, respectively. The cavity is filled with the medium of refractive index n. Right panel: resonant character of transmission for lossless cavity (Fabri-Perot interferometer).

3. Atom-cavity coupling

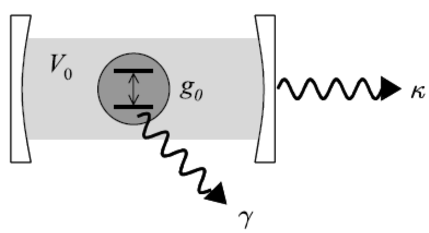

Fig. 2. Schematic of a two-level atomic system in a cavity with modal volume V_0.

The atom-cavity interaction can be simply explained by the atom absorbing photons from the cavity modes and also emitting photons into the cavity by radiative emission. Of particular interest is the case when the frequency transitions of both systems coincide. From another point of view, atom-cavity interaction can be analyzed as **interaction of two coupled (damped) oscillators**. Key parameters describing the atom-cavity interaction at resonance are:

- photon decay rate of the cavity, $\kappa = \delta\omega = \dfrac{\omega}{Q}$;

- non-resonant decay rate of the atom: non-resonant photons, photons emitted sideways $\gamma = A_{21}(1-\Delta\Omega/4\pi)/2$ (for isolated atom, pure dephasing rate is zero);
- atom-photon coupling parameter (dipole interaction of atom with vacuum field), $g_0 = \left(\frac{\mu_{12}\omega}{2\varepsilon_0\hbar V}\right)^{1/2}$

Types of coupling: strong coupling (reversible), $g_0 \gg \kappa,\gamma$; weak coupling (irreversible), $g_0 \ll \kappa,\gamma$.
Most of the results of quantum theory based on weak atom-cavity coupling can be also reproduced by classical theory of electromagnetism, where atom is considered as an oscillating electric dipole. However, this is not the case for the strong coupling where vacuum field plays important role.

a. Weak coupling regime

$g_0 \ll \kappa,\gamma$: photons are lost from the atom-cavity faster than the characteristic interaction time between the atom and cavity.
The effect of the cavity small, therefore the system can be treated by perturbation theory.
Main results: Fermi's golden rule given by eq. (1).
Contribution of cavity with a single-mode will sit in the density of states:

$$g(\omega) = \frac{2}{\pi\delta\omega_c}\frac{\delta\omega_c^2}{4(\omega-\omega_0)^2+\delta\omega_c^2}. \qquad (6)$$

Purcell factor (enhancement parameter):

$$F_P = \frac{W^{cav}}{W^{free}} = \frac{3Q(\lambda/n)^3}{4\pi^2 V}\xi^2\frac{\delta\omega_c^2}{4(\omega-\omega_0)^2+\delta\omega_c^2}. \qquad (7)$$

For the exact resonance ($\omega = \omega_0$) and with the dipole oriented along the field direction ($\xi = 1$):

$$F_P = \frac{3Q(\lambda/n)^3}{4\pi^2 V}. \qquad (8)$$

$F_P > 1$ implies that spontaneous emission rate is enhanced, $F_P < 1$ – the cavity inhibits the emission.
Another useful parameter to describe the effect of cavity is spontaneous emission coupling factor

$$\beta = \frac{W^{cav}}{W^{free}+W^{cav}} = \frac{F_p}{1+F_p}, \qquad (9)$$

which states for the fraction of the photons emitted into the cavity mode to the total number of photons emitted.
Experimental observations
First experimental demonstration of Purcell effect on atoms was done by S. Haroche et al in 1983 [2]. Specifically, the partial spontaneous emission probability on the transition 23S – 22P of Na atoms (Rydberg atoms) has been increased 530 times by using high-Q superconducting cavity tuned to resonance with a millimeter-wave transition between adjacent Rydberg states.
The first demonstrations of the Purcell effect at optical frequencies were made in the late 1980s and early 1990s by J. Heinzen et al [3], F. De Martini et al [4] in 1987 and A. M. Vredenberg et al [5] in 1993.

b. Strong coupling regime

$\sqrt{N}g_0 \gg \kappa,\gamma$: emitted photons are re-absorbed by the atom faster than they are lost from the mode.
This regime of reversible light-matter interactions bears the name of cavity quantum electrodynamics (CQED).

Main results: Jaynes-Cummings model (JCM) [6], which describes the interaction of a two-level atom with a single quantized mode of the radiation field; dressed states; vacuum Rabi splitting.
Hamiltonian of interaction in JCM:

$$\hat{H}_{JC} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_a \frac{\hat{\sigma}_z}{2} + \frac{\hbar\Omega}{2} \left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_- \right) . \quad (10)$$

where $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, $\hat{\sigma}_- = |g\rangle\langle e|$.
The electric-dipole interaction between the atom and the photon mixes the degenerate states and lifts the degeneracy (see Fig. 3, left). The expression of the energies and wavefunctions for the dressed states are the following:

$$E_n^\pm = (n+1/2)\hbar\omega \pm \sqrt{n}\hbar g_0 , \quad (11)$$

$$\Psi_n^\pm = 1/\sqrt{2} \left(|g;n\rangle \mp |e;n-1\rangle \right) . \quad (12)$$

The results of the JCM reconciles with the Rabi model considering classical field of high intensity (discussed in previous lectures) by setting $\Omega_R = 2\sqrt{n} g_0$ (factor of 2 raises from difference between standing waves in a cavity and travelling waves of a laser beam).

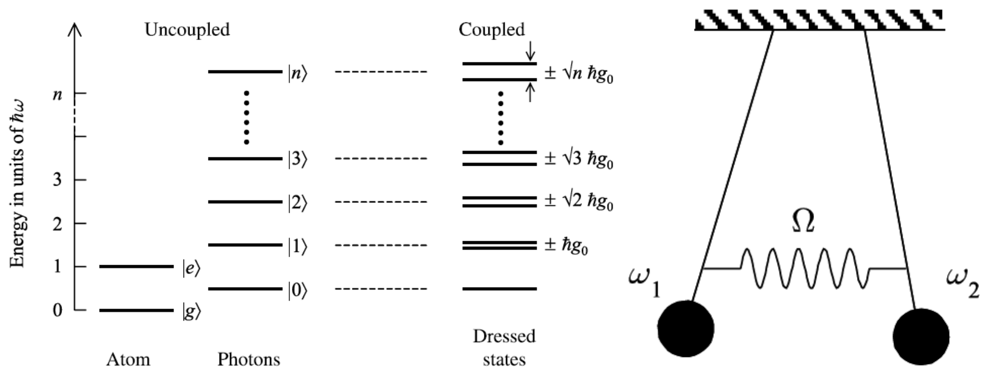

Fig. 3. Left panel: the JCM ladder. States of the coupled and uncoupled atom-photon system. Right: illustration of coupled oscillators, where $\omega_{1,2}$ are the natural frequencies and Ω is the coupling strength.

Interesting phenomenon is splitting due to vacuum field modes, so called vacuum Rabi splitting. For *N* atoms in the cavity the splitting can be expressed as

$$\Delta E_{vac}(N) = 2\sqrt{N}\hbar g_0 . \quad (13)$$

The splitting of the modes of the atom-cavity system can be given a quasi-classical explanation by considering the properties of two coupled classical oscillators shown in Fig. 3.

$$\omega_\pm = (\omega_1 + \omega_2)/2 \pm (\Omega^2 + (\omega_1 - \omega_2)^2)^{1/2} , \quad (14)$$

or simply at resonance ($\omega_1 = \omega_2 = \omega$): $\omega_\pm = \omega \pm \Omega$.

Experimental observations

Experimental observation of strong coupling interaction requires cavities with small volumes, high Q-factors; other dissipative rates due to dephasing and non-resonant emission to be minimized; cavity to support only a single mode in resonance with the atom.
Fig. 4 demonstrates the experimental arrangement and experimental results of observing the Rabi splitting.

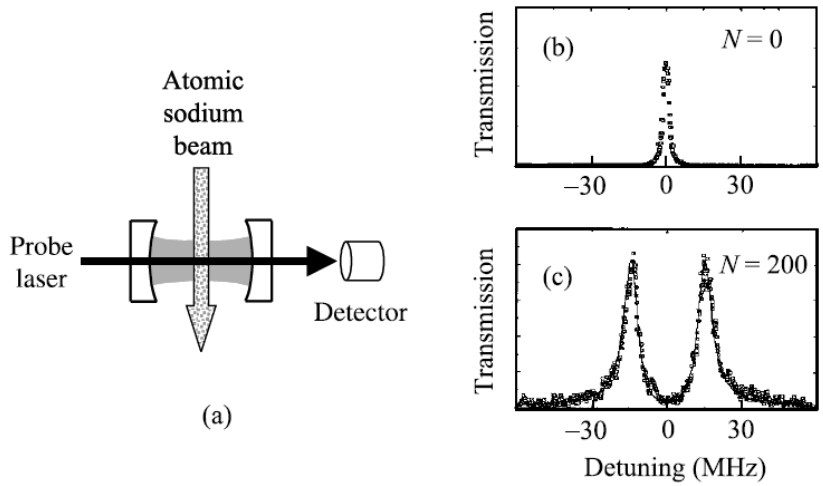

Fig. 4. Experimental demonstration of the vacuum Rabi splitting. (a) Schematic of experimental set-up. (b) and (c) correspond to the transmission spectra without and with atoms inside the cavity.

4. Applications and beyond cavities:

- low-threshold lasers – control of spontaneous emission rate to improve the performance of the laser medium;
- utilization of photonic crystals [7]/metamaterials [8] to obtain better control of spontaneous emission;
- vacuum induced transparency [9];
- development of single-photon phase gates for quantum computation [10].

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