The photon counter:

- Photomultiplier Tubes (PMT): "are extremely sensitive detectors of light in the ultraviolet, visible, and near-infrared ranges of the electromagnetic spectrum. These detectors multiply the current produced by incident light by as much as 100 million times (i.e., 160 dB), in multiple dynode stages, enabling (for example) individual photons to be detected when the incident flux of light is very low." (Wikipedia)
- Avalanche Photodiode (APD): "is a highly sensitive semiconductor electronic device that exploits the photoelectric effect to convert light to electricity. APDs can be thought of as photodetectors that provide a built-in first stage of gain through avalanche multiplication." (Wikipedia)

Photo-counting experiments: semi-classical approach where light is treated classically (electromagnetic wave) while being quantized in the detector.

Photon Flux Φ: "the average number of photons passing through a cross-section of the beam in unit time." (FQ Chapter 5.2)

$$\Phi = \frac{IA}{\hbar\boldsymbol{\omega}} = \frac{P}{\hbar\boldsymbol{\omega}}\ photons/s$$

$$I: Laser\ Intensity$$

$$A: Area\ of\ the\ beam$$

$$P: Laser\ Power$$

Average number of photocounts at an interval T by a detector with quantum efficiency η can be calculated:

$$N(T) = \eta\Phi T = \frac{\eta PT}{\hbar\boldsymbol{\omega}} = RT$$

$$R: count\ rate$$

However, due to the discreet nature of photons, perfectly coherent light incurs fluctuations over a short period of time. The distribution of photons can be described as a Poissonian model. If photons travel the length L, and assume that a positive number of photons flow through this length, the average number of photons is:

$$\bar{n} = \frac{\Phi L}{c}$$

If the length L is discretized into N segments, assuming N is sufficiently large number, the probability of finding a photon in one segment is:

$$p = \frac{\bar{n}}{N}$$

Finding n photons inside the beam of length L with N segments is:

$$P(n) = \binom{N}{n} p^n (1-p)^{N-n} = \frac{N!}{n!\,(N-n)!}\left(\frac{\bar{n}}{N}\right)^n \left(1-\frac{\bar{n}}{N}\right)^{N-n}$$

$$= \frac{1}{n!}\left(\frac{N!}{N^n(N-n)!}\right)\bar{n}^n \left(1-\frac{\bar{n}}{N}\right)^{N-n}$$

Assuming N is sufficiently large,

$$\lim_{N\to\infty}\left(\frac{N!}{N^n(N-n)!}\right) = 1$$

$$\lim_{N\to\infty}\left(1-\frac{\bar{n}}{N}\right)^{N-n} = \exp(-\bar{n})$$

Therefore,

$$P(n) = \frac{\bar{n}^n}{n!}\exp(-\bar{n}), \qquad n = 0, 1\,, 2, \ldots$$

The normalization of probability density function:

$$\sum_{n=0}^{\infty} P(n)\, n = \left(1 + \frac{\bar{n}}{1!} + \frac{\bar{n}^2}{2!} + \frac{\bar{n}^3}{3!} + \cdots\right)\exp(-\bar{n}) = \exp(\bar{n})\exp(-\bar{n}) = 1$$

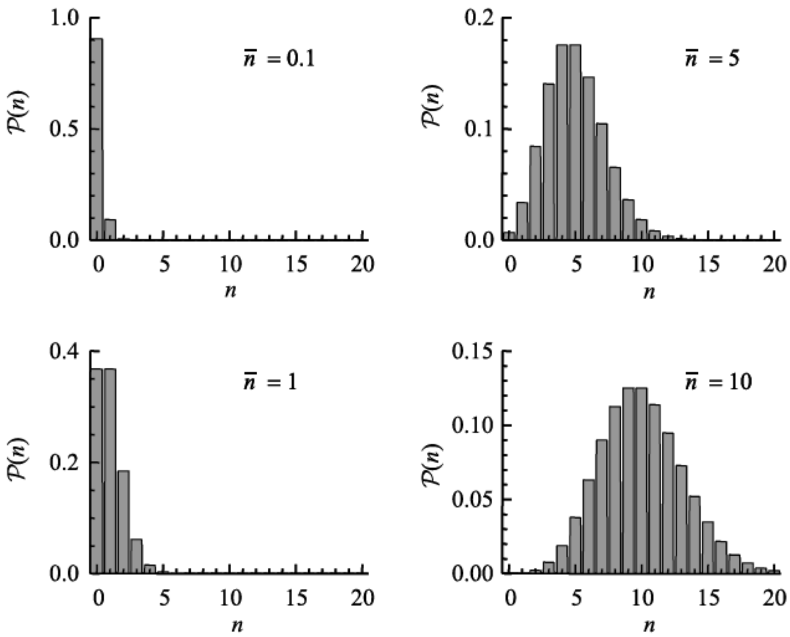

Figure 5.3 (From FQ) Poisson distributions for mean values of 0.1, 1, 5, and 10.

The variance of Poisson distributions:

$$Var(n) \equiv (\Delta n)^2 = \sum_{n=0}^{\infty} (n-\bar{n})^2 P(n)$$

$$= \sum_{n=0}^{\infty} (n^2 - 2n\bar{n} + \bar{n}^2) P(n)$$

$$= \sum_{n=0}^{\infty} n^2 P(n) - 2\bar{n} \sum_{n=0}^{\infty} nP(n) + \bar{n}^2 \sum_{n=0}^{\infty} P(n)$$

$$= \sum_{n=0}^{\infty} n^2 P(n) - \bar{n}^2$$

With further algebraic rearrangement:

$$\sum_{n=0}^{\infty} n^2 P(n) - \bar{n}^2 = \sum_{n=0}^{\infty} n(n-1)P(n) + \bar{n} - \bar{n}^2$$

By plugging in the expression for Poisson distributions in the first term on the right hand-side:

$$\sum_{n=0}^{\infty} n(n-1)P(n) = e^{-\bar{n}} \sum_{n=0}^{\infty} n(n-1)\frac{\bar{n}^n}{n!}$$

$$= e^{-\bar{n}} \left(0 + 0 + 2\frac{\bar{n}^2}{2!} + 6\frac{\bar{n}^3}{3!} + \cdots\right)$$

$$= \bar{n}^2 e^{-\bar{n}} (1 + \bar{n} + \frac{\bar{n}^2}{2!} + \cdots)$$

$$= \bar{n}^2 \sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} = \bar{n}^2$$

Therefore:

$$Var(n) \equiv (\Delta n)^2 = \sum_{n=0}^{\infty} (n-\bar{n})^2 P(n) = \bar{n}$$

Thus, this implies that the smaller the average number of photons, the more important fluctuation becomes. There are three types of light distributions.

(1) Super-Poissonian Light: characterized by the relation, $\Delta n < \sqrt{\bar{n}}$, super-Poissonian light can be partially coherent (chaotic), incoherent, or thermal light. The intensity of super-Poissonian light varies over time and is in nature unstable.

- Thermal Light (blackbody radiation): given the density of states $g(\omega)$, where L is a unit dimension and V for a unit volume for one state,

$$g(\omega) = 2 \times g(k)\frac{dk}{d\omega}$$

where the factor 2 accounts for the two photon polarizations (vertical and horizontal)

$$g(k)\,dk = \frac{4\pi k^2\,dk}{\left(\frac{2\pi}{L}\right)^3} = L^3\frac{k^2}{2\pi^2}dk = V\frac{k^2}{2\pi^2}\,dk$$

Therefore, $g(k) = \frac{k^2}{2\pi^2}$ for a unit volume. And this result leads to:

$$g(\omega) = g(k)\frac{1}{c} = \frac{2\left(\frac{\omega}{c}\right)^2}{2\pi^2 c} = \frac{\omega^2}{\pi^2 c^3}$$

The average photon energy is defined:

$$\langle E_n\rangle = \hbar\omega\langle n\rangle = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$

Here, $\langle n\rangle$ is derived from the Boltzmann statistics where $P_\omega(n) = \frac{\exp\left(-\frac{n\hbar\omega}{k_B T}\right)}{\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega}{k_B T}\right)}$.

One can arrive at the Planck's law by multiplying the average photon energy with the density of states:

$$\rho(\omega, T)d\omega = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}\frac{\omega^2}{\pi^2 c^3}d\omega$$

Using the Bose-Einstein distribution, where $P_\omega(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n$, one can arrive at the variance using the formula (N_m is the number of modes):

$$(\Delta n)^2 = \sum_{n=0}^{\infty}(n-\bar{n})^2 P(n) = \bar{n} + \frac{\bar{n}^2}{N_m}$$

- Chaotic light (FQ): the light from a single spectral line of a discharge lamp is generally called chaotic light. Chaotic light has the fluctuations in the photon number defined by:

$$(\Delta n)^2 = \langle W(T) \rangle + \langle \Delta W(T)^2 \rangle$$

where W represented the number of photon detected during an interval T,

$$W(T) = \int_t^{t+T} \eta \Phi(t')dt'$$

Here, η is the detection efficiency. The first term $\langle W(T) \rangle$ is nothing but the average photon count $\bar{n}$. If there is no intensity fluctuation, the second term $\langle \Delta W(T)^2 \rangle$ would vanish, which makes the light Poissonian.

(2) Poissonian Light: perfectly coherent light characterized by the relation, $\Delta n = \sqrt{\bar{n}}$, the intensity is constant over time.

(3) Sub-Poissonian Light: the light with photon count fluctuations defined as,

$$\Delta n < \sqrt{\bar{n}}$$

and supposedly more stable than the perfectly coherent light. Such phenomena is conceptually possible under certain controlled conditions.

The photon statistics get affected by loss from the surrounding mediums. Lossy mediums act like a beam splitter where it only allows a portion of photons to be transmitted to the detector at random. Due to this randomized "sampling" the photons would arrive at the detector in rather irregular intervals, which give rise to the photon number fluctuations. Such losses include (FQ):

(1) Inefficient quantum efficiency
(2) Surface reflection, absorption or scattering
(3) Inefficient collection optics