Notes - Lecture 2
1/15/14
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Review of Classical Optics

I. **Course Information**
II. **Intro to Classical Optics**
III. **Intro to Quantum Optics**
IV. **Properties of Light**

I - Course Information

This course will consist of five overlapping parts. Part **1** will be a *review of basic quantum mechanics* as well as *classical optics*. Part **2** will consist of several examples of *light matter interactions* such as radiative transitions in atoms, molecule and solids, as well as some of the fundamental properties of lasers. Part **3** will be on the *quantum optics of photons*. Part **4** will cover *advanced light-matter interactions* including the rabi problem, laser cooling and cavity QED. Part **5** will be on *quantum information and photonics applications* covering a wide variety of specialty topics such as various forms of quantum computation (ions, photons, superconducting qubits) photonics with nanomaterials such as graphene, and many more topics of interest. Several resources on various topics in quantum electronics, photonics, information will be posted to the course wiki site so please check regularly for more information to explore. The course grading structure will be **Homework -** (**30%), Papers and Projects (30%), Final Exam (30%) Participation/class services (10%)**

II - Into to Classical Optics

Optics is one of the oldest and most studied areas of physics, an understanding of optics is integral into almost every other aspect of physics. Optics can be split up into two main branches, Classical optics (consisting of Geometric optics and wave optics) and Modern optics (consisting of Quantum optics, nonlinear optics, and statistical optics). Classical optics is based on relative few simple principles such as which can be used to derive several very powerful results, it is actually quite remarkable how many phenomena can be described by classical optics alone.

One of the most astounding experimental results from classical optics and astronomy was the measurement of the speed of light. Nowadays it is somewhat taken for granted how well we know the speed of light 299,792,458 m/s, but several hundred years ago in the absence of high frequency oscillators, clocks, and modern precision measurement equipment the idea that someone could something so fast to any degree of accuracy at all seems quite daunting. Historically the first measurement of the speed of light was performed by Ole Roemer in 1676. Roemer used two astronomical occurrences to make his measurement of the speed of light,

the orbit of the earth around the sun and the orbit of jupiter's moon Io. As Io orbits Jupiter when it passes behind jupiter as seen from earth an eclipse is seen, (sight of Io from earth is eclipsed or blocked out by jupiter until it passes back from behind it. If the speed of light were infinite you would expect to see Io the instant it finished traversing behind jupiter, regardless of how far away you are. Assuming Io is undergoing a roughly circular orbit you would thus expect to always measure the same eclipse duration for Io passing behind jupiter regaurdless of how far away jupiter is from the earth. Roemer measured this eclipse duration and actually found that it varies depending on the date that it is measured. This could thus be attributed to the difference in distance the light had to travel to earth from Io occurring from the movement of the earth about the sun (when the earths orbit is closest to jupiter the time is minimum when the earth orbit is furthest away from jupiter the delay is maximum). Using this change in delay in the eclipse and the appropriate change in earth to jupiter distances known at the time Roemer was able to deduce a speed of 211,000 km/s (about 0.7 times the modern accepted value, Not bad for the 1600's!)

Geometric or Ray optics is based on Newton's corpuscular or particle theory of light. All of geometric optics can be derived from Fermat's principle (1657) which states that when traveling between two point light takes the path which requires the least time. This simple principle give rise to explanations for many common optical phenomena and allowed the derivation of the law of reflection and snell's law of refraction. Simple geometric optics provides an adequate framework to mathematically describe the optical properties of simple lenses, beam splitters, and mirrors to a good degree of accuracy

Wave optics is derived from Huygen's principle (1678) which predicts that light is a wave and each point of the wave front acts as a source for a new spherical wave. Using this new axiom Huygens was able to accurately account for reflection and refraction much like geometric optics. In (1816) Fresnel incorporated the notion of wave-superposition interference with huygens principle or the Hyugens-Fresnal principle that wave optics was able to account for the phenomena of interference and diffraction patterns, both of which could not be explained by geometric optics alone. It was until Maxwell (1861) unified the Electric and Magnetic fields in his famous Maxwell's equations and derived the wave equations for a propagating electromagnetic wave that classical Wave optics really started to reach its full potential.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

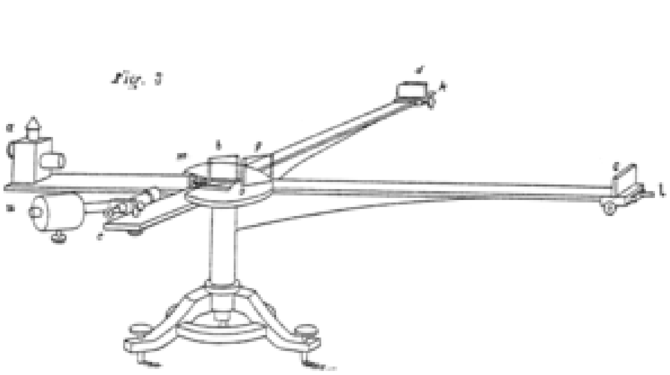
$$\nabla \times \vec{B} = \frac{\vec{J}}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$$

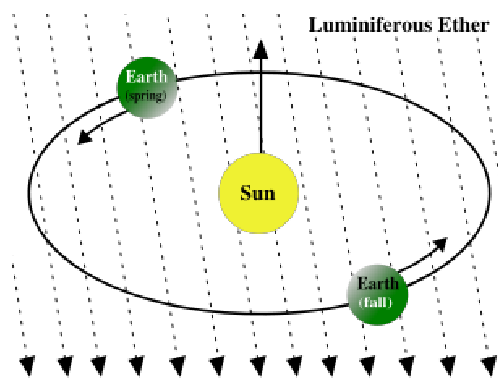
In our modern era of GR, quantum mechanics, and string theory, the impact of Maxwells equations and the merit of the unification of the electric and magnetic fields is often lost, and taken for granted. Nowadays we are taught from the very beginning of our physics education that electric and magnetic phenomena are deeply intertwined aspects of the same phenomena, but at the time of it's discovery it was a truly monumental feat, when Einstein was a young physicist he dreamed of one day having an impact upon physics the way the Maxwell had.

One important peculiarity from maxwell's equations was the fact the light was predicted to always travel at a fixed speed 'c'. This caused great difficulty because at the time it was thought that for this to be true there had to be one absolute newtonian reference frame which was comprised of some medium called the ether which all other objects moved through. Before the description of electromagnetic waves several other mechanical wave phenomena had been described and had been referenced as mechanical waves traveling on a given media such as sound waves in air or ripples in water, the existence of the ether was a somewhat natural assumption following from experience with other wave phenomena.

In an attempt to study this peculiar ether, Michelson and Morley (1887) devised an experiment to measure the relative motion of the earth through the ether using an interferometer. The apparatus started with a light source which sends out an incident beam in lets say the -x direction The beam is then incident upon a beam splitter which would separate the incoming beam into two separate beams, one continuing in the incident beam direction (-x) and the other in a perpendicular direction (y). Both of the these two new beams would travel down the "arms" of the interferometer until reaching mirrors at the end of each respective arm and being reflected back towards the beam splitter. These beams would then recombine at the beam splitter and a final transmitted beam would be sent propagation towards a viewing detector in the -y direction.

(images from wiki http://en.wikipedia.org/wiki/Michelson%E2%80%93Morley_experiment)





The principal of the experiment was to detect relative motion of the earth to the ether. If the ether existed in some absolute rest frame through which all of matter moved freely, and light propagated through this ether at uniform speed c in all directions, one would expect to observe a directional difference in the observed speed of light due to the earth's motion relative to the either. In the interferometer, if one arm of the interferometer is aligned parallel to the earth-ether motion and the other is perpendicular, one would predict that the light beam being reflected back

and forth through the arm parallel to the earth-ether movement would arrive at the detection later than the beam traveling through the perpendicular arm. This would give rise to an observable fringe shift in the interference pattern as the angle of the arms is rotated through the relative direction of earth-ether motion. Their experiment was famously a null and would later guide Einsteins work towards his theory of special relativity (1905) which would largely complete the underlying framework of classical wave optics.

III - Intro to Quantum optics

In the early twentieth century our understanding of the nature of light underwent significant advances due to the onset of quantum mechanics. The ultraviolet catastrophe was an outstanding problem from thermodynamics resulting from a failed attempt to describe blackbody radiation. Around 1900-1905 Lord Rayleigh and Sir James Jeans developed the Rayleigh-Jeans Law which predicted that a blackbody should have a spectral radiance proportional to temperature divided by wavelength to the 4th power. One consequence of this theory was that it gave rise to the so called the ultraviolet catastrophe which predicted a blackbody emitting divergent amount of energy at higher frequency radiation. Max Planck discovered the solution to the problem by quantizing the amount of energy of a emitted by a mode in a thermal system ($E = nhf$). The full significance of this wasn't realized until later but this solved the ultraviolet catastrophe by associating a cut off frequency proportional to the temperature of the black body which exponentially quenched the higher frequency modes due to having insufficient thermal energy to emit a whole photon.

The second experiment demonstrating the quantum nature of light was the photoelectric effect which was explained by Einstein in 1905. As an example setup for a photoelectric experiment consist of a light source at frequency f, and two closely spaced parallel electrodes or metal plates, (call them ‘A’ and ‘B’), connected by a wire. If the light source illuminates plate ‘A’ for frequencies higher than a certain material dependent threshold frequency electrons will be ejected from plate ‘A’. Some of these electrons will strike plate ‘B’ causing a current to flow between the two plates. For frequencies below the threshold frequency no current will flow no matter what the intensity of the light source is. For frequencies above the threshold as the intensity of the light is increased the current will also increase. Next consider the same apparatus running with incident light above the threshold frequency causing a current to flow, but this time with a voltage source in between the two plates. If a negative bias voltage is applied to plate B and increased to a certain level, eventually it will be seen that the current from electrons striking plate B will go to zero, upon reaching the zero current point increasing the intensity will no longer increase the current. If the frequency of the light source is increased a current will again be observed and if the intensity of this higher frequency light source is increased once again the current will be seen to increase.

Einstein was able to successfully explain these observations by introducing the notion that light is composed of discrete packets of energy or photons containing energy hf. The characteristic threshold frequency of the metal plate represents the ionization energy of the metal, the energy to remove one electron from the negative coulomb potential. Photons of

energy less than this energy are unable to “free” any electrons no matter how much total energy is incident from the light source. The energy of the individual photon has to be greater than or equal to the ionization energy. For a light source of frequency above the threshold frequency increasing the intensity causes an increase in current, this can be attributed to more electrons being liberated. As the bias voltage is applied this increases the amount of kinetic energy an electron must have to travel from plate A to plate B. the quenching effect observed at sufficiently negative voltage can be attributed to the ionized electrons not having enough kinetic energy to traverse the potential difference between ‘A’ and ‘B’. As the frequency is increased sufficiently the electrons will once again have enough energy to travel from “A’ to ‘B’ allowing a current to flow.

IV - Properties of Light

Throughout this course a basic understanding of several properties of light will be needed to delve deeper into light matter interactions. As mentioned before light is seen to behave as both a wave and a particle. Like all wave light (electromagnetic waves) are described by an amplitude, frequency, phase, and propagation speed ‘c’ = frequency times wavelength. Light carries a momentum h bar k and contains an energy hf. One of the most important properties of light is it’s polarization, eg the alignment of its electric and magnetic fields. The energy flux of electromagnetic waves is giving by the poynting vector S = (E X B) * (1/ the vacuum permeability constant). A wave propagating in the z direction will therefore have electric and magnetic fields aligned in the x-y plane, perpendicular to the direction of propagation and each other. The simplest example of polarization is linear polarization with the electric and magnetic fields constrained to one component aligned to a linear axis eg E polarized in the x direction oscillating between +x and - x, and the magnetic field polarized in the y direction oscillating between +y and -y. Another more complicated example is circular polarization where the electric and magnetic fields have components in say both the x and y axis with one lagging behind the other by a phase factor of pi/2. as the wave propagate along the z axis the electric and magnetic field polarizations will appear to rotate, clockwise or counterclockwise depending on the phase relation. A third polarization of light is unpolarized, which is simply and even combination of both x and y polarization. If one were to measure the transmitted intensity of unpolarized light propagating in the z direction after it passes through a linear polarizer aligned in the x-y plane, no matter what angle it was aligned to in that plane, half of the incident light will be observed to be transmitted through the polarizer.