Photon Entanglement and Teleportation

- Entangled States

A multi-particle system is described as being in an entangled state if its wave function cannot be factorized into a product of the wave functions of the individual particles. The wave function has to be written in the forms:
Perfect positive correlation:

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0_1, 0_2\rangle \pm |1_1, 1_2\rangle\right),$$

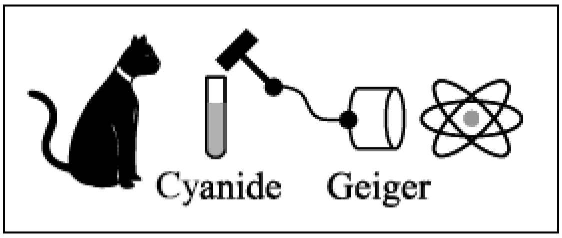
And Perfect negative correlation:

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0_1, 1_2\rangle \pm |1_1, 0_2\rangle\right),$$

In both cases a measurement on one photon allows us to predict the result of the measurement on the other with 100% certainty.
Entangled states can be produced by using a CNOT gate:

$$|1\rangle + |0\rangle \otimes CNOT|0\rangle = |00\rangle + |11\rangle$$

Two-particle photon states with time or momentum entanglement can also be generated, and entangled states involving three or more particles have many interesting properties. Large number of "bodies" entanglement is still a challenging topic.
The Schrodinger cat paradox illustrates the concept of entangled states in a graphic way by considering the state of a live cat put into a sealed box containing a radioactive atom. The box also contains a devious mechanism such that the decay of the atom triggers a device to smash a sealed flask of poison, thereby killing the cat.



We can write the wave function of the system in the form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{live}, 1\rangle + |\mathrm{dead}, 2\rangle\right)$$

This seems to imply that we have a state inside the box where the cat is both dead and alive at the same time. On opening the box, we would find the cat dead or alive with probability equal to 50%.
In fact, large systems consisting of many particles will lose their quantum coherence because of the noisy macroscopic environment. Only at the microscopic level, quantum coherence of isolated atoms and photons will play an important role.

- Generate Entangled States

Entangled states can be generated by a three level system. When the system drops from initial state to middle state, a photon is emitted. When the system drops from middle state to final state, another photon is emitted. These two photons form a photon pair.

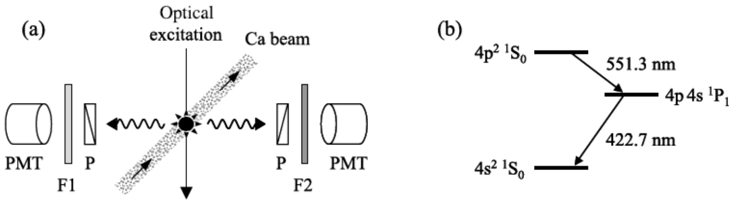
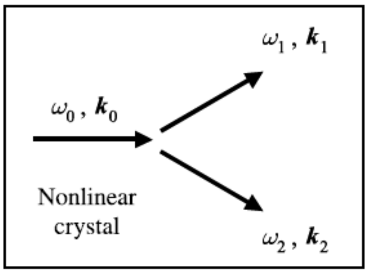


Fig. 14.3 Correlated photon pair generation by atomic cascade in calcium. (a) Experimental arrangement employing two linear polarizers (P) and photomultiplier tube (PMT) detectors. (b) Atomic level scheme. The narrow-band interference filters F1 and F2 used in the experiment were chosen to select the photons at 551.3 and 422.7 nm, respectively. (After C. A. Kocher and E. G. Commins, *Phys. Rev. Lett.* **18**, 575 (1967).)

The initial and final states are both J = 0 states. This demands that the photon pairs emitted has no net angular momentum. In addition, the rotational invariance of J = 0 states, and the fact that the initial and final levels are both of the same even parity, requires that the photon pairs have the polarization correlation properties required for the EPRB experiments.

Entangled states can also be generated with the help of nonlinear optics device. A single photon from a pump laser at angular frequency ω0 is converted into a pair of signal and idler photons at angular frequencies ω1 and ω2 by nonlinear crystal.



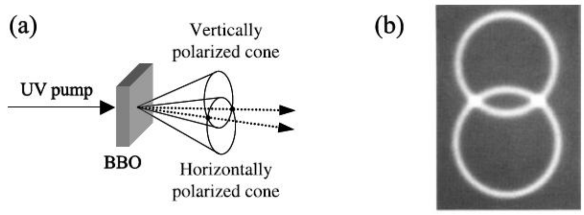
According to conservation of energy and momentum, we have:

$\omega_1 + \omega_2 = \omega_0$

$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_0$

When both equations are satisfied, they are called phase-matching conditions. The down-conversion process is called degenerate when $\omega_1 = \omega_2 = \omega_0/2$.

There are only two types of phase matching. In type-I phase matching the polarizations of the down-converted photons are parallel to each other and orthogonal to the pump photon, while in type-II phase matching the down-converted photons have orthogonal polarizations.



Entangled photon pairs can be generated by degenerate down-conversion with type-II phase matching. The phase matching requirements determine that the down-converted photons emerge in cones of opposite polarization, leading to a double ring pattern with two intersection points. This system produces states of the type:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow_1, \updownarrow_2\rangle + \mathrm{e}^{\mathrm{i}\phi}|\updownarrow_1, \leftrightarrow_2\rangle\right)$$

- Single Photon Interferometer

An entangled photons pair is generated by the nonlinear crystal. The path difference between the signal and idler photon can be adjusted. The total signal on detectors D1 and D2 would be constant, but the magnitude of the signal on the individual detectors would oscillate in anti-phase as BS is translated.

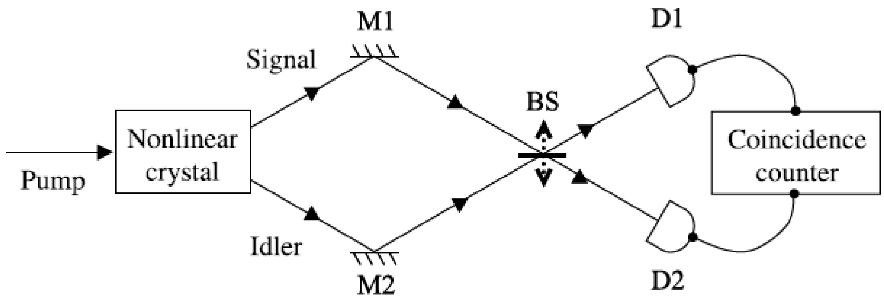


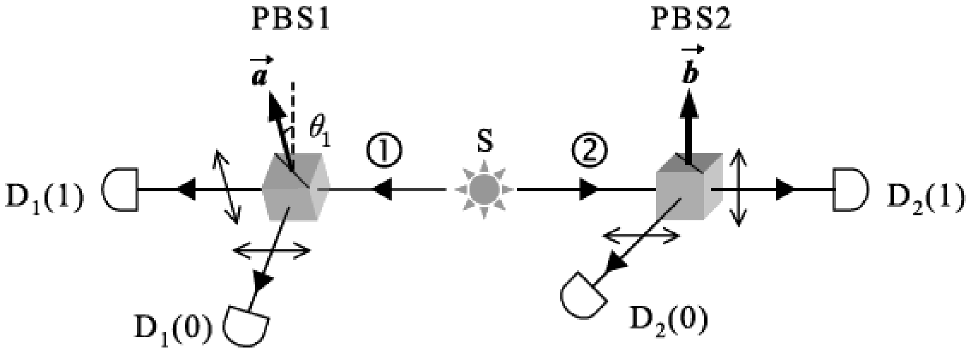
Fig. 14.6 Experimental arrangement for demonstrating single-photon interference effects using correlated photon pairs. M1 and M2 are mirrors, D1 and D2 are single-photon counting detectors, and BS is a 50 : 50 beam splitter. The path difference between the signal and idler beams can be adjusted by translating BS up and down. (Adapted from C. K Hong, Z. Y. Ou, and L. Mandel, *Phys. Rev. Lett.* **59**, 2044 (1987).)

When the path lengths of the beams are identical, the two photons arrive at the beams splitter at the same time and interfere. When single photons interfere at a 50 : 50 beam splitter, destructive interference prevents the possibility that the two photons go to different output ports, and both photons therefore emerge at the same output. Hence the only possible results are that both photons go to one detector.
This kind of set up is called a Hong–Ou–Mandel interferometer.

- Bell's theorem/inequality

Bell's theorem states that the inequality is always obeyed if the LHV picture of the microscopic world is correct. Quantum mechanics, by contrast, predicts violations of Bell's inequality. In order to perform a measurement of Bell's inequality on a pair of

correlated photons emitted from a source S, we allow the axes of the two polarizing beam splitter cubes to be different.



$\mathcal{P}_{11}(\theta_1, \theta_2)$ is the probability that $D_1(1)$ fires and $D_2(1)$ fires,
$\mathcal{P}_{10}(\theta_1, \theta_2)$ is the probability that $D_1(1)$ fires and $D_2(0)$ fires,
$\mathcal{P}_{01}(\theta_1, \theta_2)$ is the probability that $D_1(0)$ fires and $D_2(1)$ fires,
$\mathcal{P}_{00}(\theta_1, \theta_2)$ is the probability that $D_1(0)$ fires and $D_2(0)$ fires.

Consider the problem according to quantum mechanics approach first. We can get the following results:

$$\mathcal{P}_{11}(\theta_1, \theta_2) = \frac{1}{2}\cos^2(\theta_1 - \theta_2),$$

$$\mathcal{P}_{10}(\theta_1, \theta_2) = \frac{1}{2}\sin^2(\theta_1 - \theta_2),$$

$$\mathcal{P}_{01}(\theta_1, \theta_2) = \frac{1}{2}\sin^2(\theta_1 - \theta_2),$$

$$\mathcal{P}_{00}(\theta_1, \theta_2) = \frac{1}{2}\cos^2(\theta_1 - \theta_2).$$

Then consider the LHV approach. We get another set of results:

$$\mathcal{P}_{11}(\theta_1, \theta_2) = \frac{1}{2}(\sin^2\theta_1 \sin^2\theta_2 + \cos^2\theta_1 \cos^2\theta_2),$$

$$\mathcal{P}_{10}(\theta_1, \theta_2) = \frac{1}{2}(\sin^2\theta_1 \cos^2\theta_2 + \cos^2\theta_1 \sin^2\theta_2),$$

$$\mathcal{P}_{01}(\theta_1, \theta_2) = \frac{1}{2}(\cos^2\theta_1 \sin^2\theta_2 + \sin^2\theta_1 \cos^2\theta_2),$$

$$\mathcal{P}_{00}(\theta_1, \theta_2) = \frac{1}{2}(\cos^2\theta_1 \cos^2\theta_2 + \sin^2\theta_1 \sin^2\theta_2).$$

These two sets of results are different. The quantum model predicts the correct outcome.

$$S = E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2'),$$

$$E(\theta_1, \theta_2) = \mathcal{P}_{11}(\theta_1, \theta_2) + \mathcal{P}_{00}(\theta_1, \theta_2) - \mathcal{P}_{10}(\theta_1, \theta_2) - \mathcal{P}_{01}(\theta_1, \theta_2),$$

Then Bell inequality holds for all possible LHV theories.

$$-2 \leq S \leq 2,$$

On the other hand, quantum model violate Bell inequality.

- Teleportation

The basic idea of teleportation is to transfer the quantum state of one photon to another that is physically separated from it. There are some general principles of its operation

1. The **quantum no-cloning theorem** says that it is not possible to clone the original photon. The input photon must therefore either be destroyed or lose its initial state in an irretrievable way.
2. The general theory of quantum measurement implies that the fidelity between the output and input wave functions is degraded in proportion to the amount of information gleaned about $|\psi\rangle$ within the teleportation machine. Perfect fidelity can only be achieved when the machine retains no information whatsoever about the unknown quantum state.
3. No *matter* is teleported between the input and output, only *quantum information*.
4. Relativity tells us that we cannot transmit information faster than the speed of light. Therefore, teleportation cannot be used for superluminal information exchange.

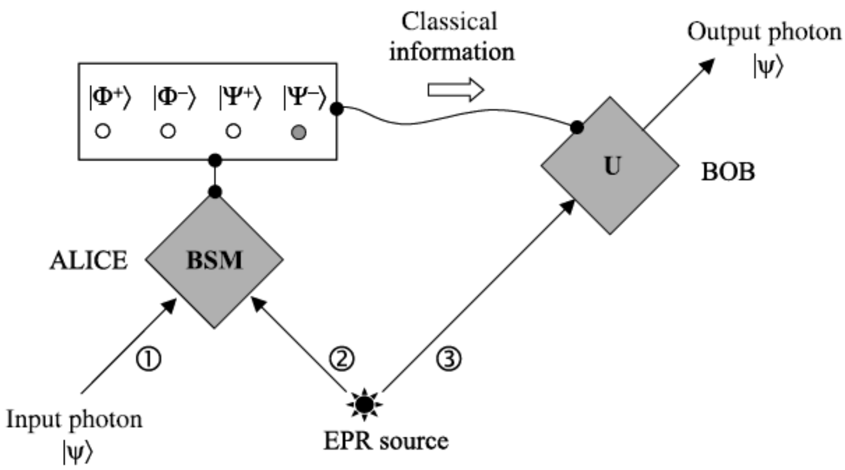


Fig. 14.11 Schematic diagram of an arrangement for photon teleportation. Photon 1 is the input photon whose quantum polarization state $|\psi\rangle$ is to be teleported. Photons 2 and 3 comprise a correlated pair from an EPR source. Alice receives photons 1 and 2 and makes a Bell state measurement (BSM) on them. Bob receives photon 3 and makes a unitary operation (U) on it according to the result of Alice's measurement, which is communicated via a classical channel. Photon 3 then emerges in the same quantum state $|\psi\rangle$ as photon 1.

Three photons are required in this process. Photon 1 is the input photon, which is in an arbitrary polarization states. Photons 2 and 3 are emitted by an EPR source. They

are in entangled state. Photons 1 and 2 are sent to Alice and photon 3 to Bob. Alice performs a 'Bell-state measurement' (BSM) on her two photons giving one of four possible results. She communicates this result to Bob by a classical channel, and Bob then performs a unitary operation U to photon 3 depending on the information he has received from Alice. The output state of photon 3 is identical to that of the original photon. In this process, we destroy the input photon and create an output photon identical to the input.