Laser Fundamentals

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Lecture Notes 9, ECE695 Introduction to Quantum Optics and Quantum Photonics

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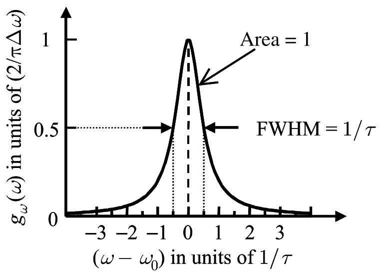
1. Spectral Line Shape

The spectral line shape is defined by a function $g_\omega(\omega)$ that describes the energy exchange between matter (e.g. atoms, ions, molecules) and light in the frequency domain. The parameters of this function are the line position, the maximum height and the width at the half maximum. Several processes including natural, collisions or Doppler effects, could characterize this function.



$$\int_0^\infty g_\omega(\omega)\, d\omega = 1$$

Even in the perfect energy transition from an exited state to ground state there is a broadening process due to spontaneous decay rate and the related lifetime of the exited state. We can probe this using the uncertainty principle and the plank relation between energy and frequency. This process is called natural broadening and is homogeneous. This means that all the atoms in a system have equal probability to radiate from a specific level.

$$E = \hbar\omega$$

$$\Delta E \Delta t \geq \hbar$$

$$\Delta\omega = \frac{\Delta E}{\hbar} \geq \frac{1}{\tau}$$

The homogeneous broadening processes are characterized by a Lorentzian distribution, where $\Delta\omega$ is defined directly by of the inverse lifetime of the excited state.

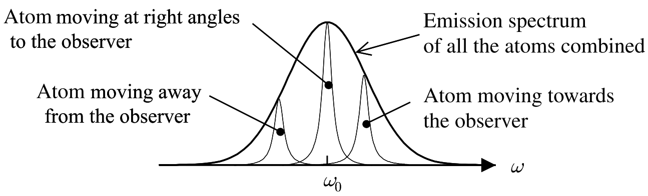
$$g_\omega(\omega) = \frac{\Delta\omega}{2\pi} \frac{1}{(\omega - \omega_0)^2 + (\Delta\omega/2)^2}$$

The second kind of broadening process is due to the collisions between atoms and is also homogeneous broadening. In this case the lifetime includes the collisions or pressure component in the following way:

$$\tau_{\text{collision}} = \frac{1}{\sigma_s P} \sqrt{\frac{\pi m k_B T}{8}}$$

where k_B is the Boltzmann constant, T is the temperature, P is the pressure and σ_s is the collision cross-section.

The third process involving broadening is due to the Doppler effect. This effect originated by the random velocity of atoms in a gas which generates Doppler shift in the observed frequencies. This process has in consequence an inhomogeneous broadening due to the probability Maxwell-Boltzmann distribution of the velocities of the atoms $N(u_x)$ forming a Gaussian line shape.



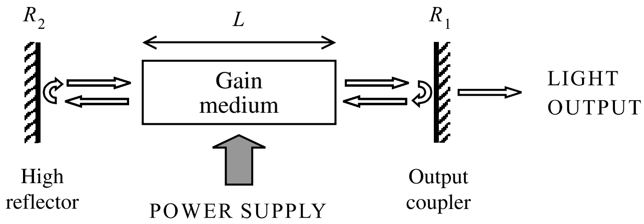
$$N(u_x) = N_0 \sqrt{\frac{2k_B T}{\pi m}} e^{\left(\frac{-m u_x{}^2}{2k_B T}\right)}$$

$$g_\omega(\omega) = \frac{c}{\omega_0} \sqrt{\frac{m}{2\pi k_B T}} e^{\left(\frac{-mc^2(\omega-\omega_0)^2}{2k_B T \omega_0{}^2}\right)}$$

2. Laser Oscillation

A laser is a special source of light with high spatial and temporal coherence, which in consequence leads us to a very narrow beam and very narrow spectrum respectively. This coherence is thanks to an optical amplification process, in which an excited energy state of an electron interacts with a photon generating a new photon with the same frequency, phase,

polarization and direction of the incident photon, and moving the electron to a lower energy state. This process is called stimulated emission and was theoretical predicted by A. Einstein in 1916. The most common structure of a laser includes a highly reflective mirror, an output coupler that let escape a portion of the internal light, a gain medium in which the stimulated emission process takes place, and a source that excites the atoms or molecules of the gain medium.



We can characterize the gain of the medium a function of angular frequency $\gamma(\omega)$ and we can quantify the amplification process by:

$$\frac{dI}{dz} = \gamma(\omega)I(z)$$

where I is the optical intensity of the beam.

$$I(z) = I_0 e^{\gamma z}$$

In order to have a laser operation we need that the stimulated emission rate dominates over the absorption rate given by

$$B_{21}^{\omega} N_2 u(\omega) > B_{12}^{\omega} N_1 u(\omega)$$

$$N_2 > \frac{g_2}{g_1} N_1$$

However, the last inequality cannot be valid if the system is in thermal equilibrium, which leads us to a special condition called population inversion given by

$$\Delta N = N_2 - \frac{g_2}{g_1} N_1$$

necessary to have a medium with optical gain. This condition is usually achieved by injecting the enough pump energy to promote a large amount of atom to an excited state. With some algebraic manipulation we can calculate the gain of the medium using

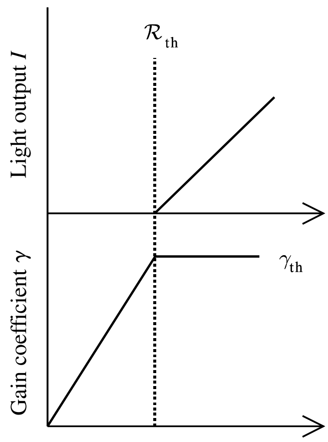
$$\gamma(\omega) = \frac{\lambda^2}{4n^2\tau}\Delta N g_\omega(\omega)$$

Now, to have a steady state laser oscillation the optical round-trip gain need to exactly balance the round-trip losses $R_1R_2\xi$ in the following way:

$$R_1R_2\xi e^{2\gamma L} = 1$$

$$\gamma = -\frac{1}{2L}\ln(R_1R_2) - \frac{1}{2L}\ln(\xi)$$

The pump power necessary to have enough optical gain to compensate the total round-trip losses in is called the pump power threshold with the correspondent gain threshold. When pump power is increased over the threshold the additional pump power is directly and proportionally converted into output power through the stimulated emission process.



3. Laser Modes

The properties of the cavity and the boundary conditions determine the shape of the output laser beam. The first mode to analyze is the laser transverse mode, which describes the intensity profile of the laser in a transverse cross section. This mode is describe by:

$$E_{mn}(x,y) = E_0 H_m\left(\frac{\sqrt{2}x}{w}\right) H_n\left(\frac{\sqrt{2}y}{\omega}\right) e^{-\frac{x^2+y^2}{w^2}}$$

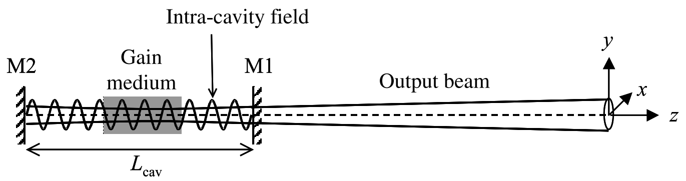
$$H_0(u) = 1$$

$$H_1(u) = 2u$$

$$H_2(u) = 4u^2 - 2$$

where the functions $H_m(u)$ are called Hermite polynomials and w describes the wide of the beam. The most common and useful transverse mode is E_{00} also called TEM_{00} due to it has the lowest loss level can be achieved.

$$E_{00}(x, y) = E_0 e^{-\frac{x^2+y^2}{w^2}}$$



The longitudinal modes of the laser are standing wave patterns formed by the electric filed inside the cavity due to the boundary conditions. These longitudinal mode most fulfill the phase condition in which the round trip phase shift is equal to an integer number m times 2π.

$$\frac{2\omega l}{c} = 2m\pi$$

This means that only the laser is only allowed to oscillate at discrete angular frequencies given by

$$\omega_m = \frac{m\pi c}{l}$$

A comparison between the cavity loss and gain in frequency domain and the location of the longitudinal modes give us a picture of the mode that will operate in the laser. Single-mode and multi-mode operation are the cases when only one or multiple longitudinal modes can be amplified respectively. The multi-mode operation can be observed in the case of inhomogeneous broadening. When all the phases of the modes in multimode operation are locked or aligned, the output of the laser is a train of pulses. In this case, the separation of the

longitudinal modes is inversely proportional to the separation of the pulses. This kind of operation is called mode-lock.

4. Laser Gain Materials

- Dopped solid-state laser. The solid host materials are usually doped with an impurity such as chromium, neodymium, erbium or titanium ions. Typical hosts include YAG (yttrium aluminium garnet), YLF (yttrium lithium fluoride), sapphire (aluminium oxide) and various glasses. Examples of solid-state laser media include Nd:YAG, Ti:sapphire, Cr:sapphire (usually known as ruby), Cr:LiSAF (chromium-doped lithium strontium aluminium fluoride), Er:YLF, Nd:glass, and Er:glass. Solid-state lasers are usually pumped by flashlamps or light from another laser.
- Gases, such as carbon dioxide, argon, krypton and mixtures such as helium-neon. These lasers are often pumped by electrical discharge.
- Diode lasers are an electrically pumped semiconductor laser in which the active medium is formed by a p-n junction of a semiconductor diode similar to that found in a light-emitting diode. Diode lasers are typically very small, and can be pumped with a simple electric current, enabling them to be used in consumer devices.

5. References

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[4] https://en.wikipedia.org/wiki/Laser

[5] https://en.wikipedia.org/wiki/Stimulated_emission

[6] https://en.wikipedia.org/wiki/Laser_construction