Lecture 3 --Review of Quantum Physics & Basic AMO Physics

Version #1 -- 2/9/2014

I Some important model systems in QM

i) 2-state QM (d=2, simplest hilbert space)
spin-1/2 (↑ & ↓)
photon 2-polarization (⇔ & ⇕)
ii) Particle in box /1D Schrodinger
iii) Harmonic oscillator
iv) Hydrogen atom → AMO physics
v) Periodic potential → solid state physics

II Foundations of QM: early milestones

i) Photon (Planck blackbody radiation)
E=hf=$\hbar\omega$

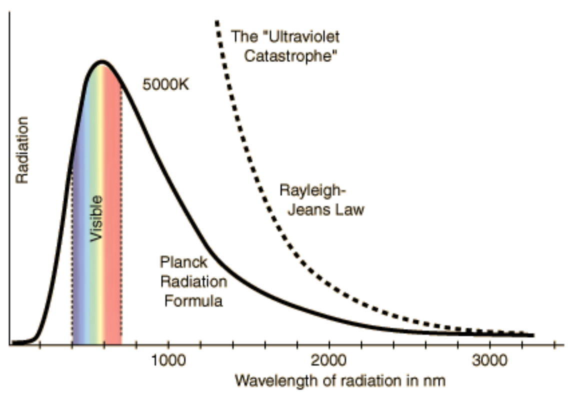


Fig 3.1

Total energy per unit volume in the cavity in ν and ν+dν
Rayleigh – Jeans formula: $u(\nu)d\nu = \varepsilon G(\nu)d\nu = (8\pi kT/c^3)\nu^2 d\nu$
Plank radiation formula: $u(\nu)d\nu = (8\pi h/c^3)(\nu^2 d\nu)/(e^{h\nu/kT} - 1)$

ii) Photo-electric effect & Compton scattering (light matter interaction)
… *PE can measure Planck constant*
Setup of Photo-electric effect experiment: setup and Experiment Results

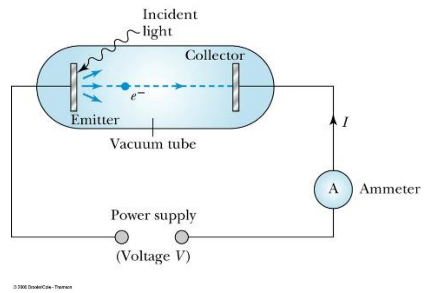
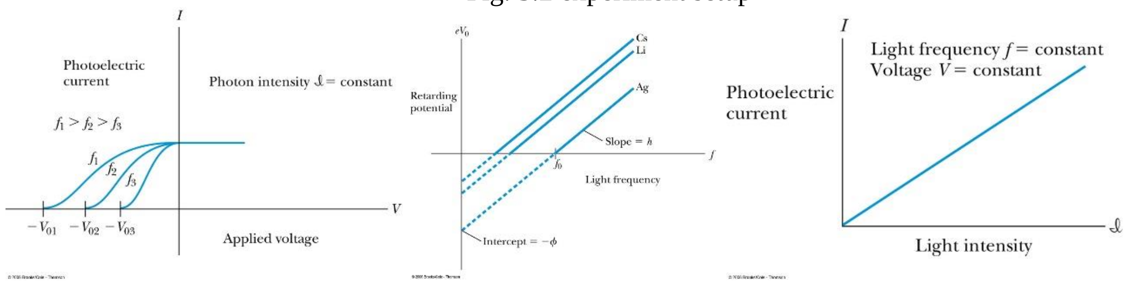


Fig. 3.2 experiment setup



(a) (b) (c)

Fig. 3.3 Experiment results

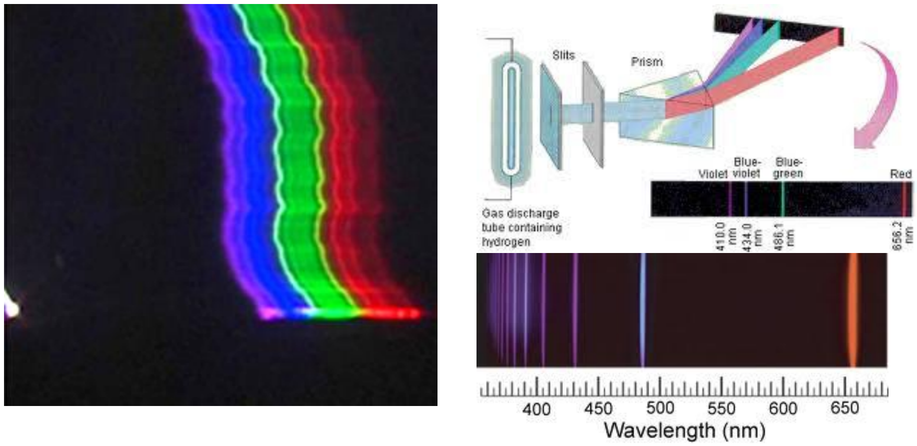
Einstein's Theory

- Conservation of energy yields:
  Energy before (photon) = energy after (electron)
  $$hf = \phi + \text{K.E. (electron)}$$
  where ϕ is the work function of the metal.
  Explicitly the energy is $hf = \phi + \frac{1}{2}mv_{\max}^2$
- The retarding potentials measured in the photoelectric effect are the opposing potentials needed to stop the most energetic electrons.
  $$eV_0 = \frac{1}{2}mv_{\max}^2$$

Quantum Interpretation

- The kinetic energy of the electron does not depend on the light intensity at all, but only on the light frequency and the work function of the material.
  $$\frac{1}{2}mv_{\max}^2 = eV_0 = hf - \phi$$
- Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope *h*, the same constant found by Planck.
  (fig. 3.3 (b))
  $$eV_0 = \frac{1}{2}mv_{\max}^2 = hf - hf_0 = h(f - f_0)$$
- From this, Einstein concluded that light is a particle with energy:
  $$E = hf = \frac{hc}{\lambda}$$

iii) Spectroscopy (atomic physics/astronomy) --- start from hydrogen --- Bohr's model (atom/matter quantized levels)



(a) (b)

Fig. 3.4 (a) Spectrum of Ball Lightning (b) Hydrogen Spectrum

External link:
http://physics.aps.org/articles/v7/5

Observation of the Optical and Spectral Characteristics of Ball Lightning
Jianyong Cen, Ping Yuan, and Simin Xue
Phys. Rev. Lett. **112**, 035001 (2014)

III Concepts & theory

i) Matter wave; wave-particle duality (De Broglie)

Energy: $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{{m_0}^2 c^4 + P^2 c^2} = \sqrt{{E_0}^2 + P^2 c^2}$

$$m_0 = rest\ mass$$
$$E_0 = rest\ energy$$

Momentum: $P = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = h/\lambda = h/(mv)$

Comparing to Photon (massless particle): E=hf=pc => p=hf/c=h/λ

P=h/λ is true for all particles

Wave Optics: diffraction

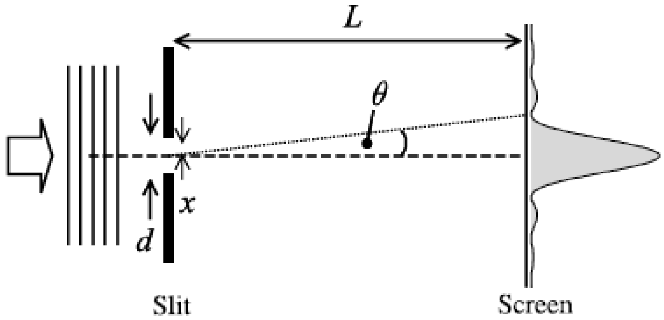


Fig. 3.5 When the distance L between the slit and screen is larger than the **Rayleigh Distance** (d^2/λ) , the diffraction pattern is said to be in the **far-field (Fraunhofer) limit**. On the other hand, when $L \lesssim d^2/\lambda$, we are in the **near-field (Fresnel) regime**. In what follows, we consider only Fraunhofer limit

Diffraction pattern: $I(\theta) \propto (\frac{sin\beta}{\beta})^2$

Resolution: $\sin\theta_{min} = \pm\frac{\lambda}{d}$

resolution for circular aperture: $\sin\theta_{min} = 1.22\frac{\lambda}{d}$

ii) Uncertainty principle (Heisenberg)

- Fourier transform picture

Heisenberg Uncertainty Principle: $\Delta x \Delta P_x \geq \hbar/2$

IV Theory & Formulation

i) Schordinger equation/wavefuction

$$\mathcal{P}(\boldsymbol{r},t)\,\mathrm{d}V = |\Psi(\boldsymbol{r},t)|^2\,\mathrm{d}V.$$ -- probability density

The equation of motion of the wave function is given by the **Schrödinger equation**:

$$\hat{H}\,\Psi(\boldsymbol{r},t) = \mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t), \tag{3.2}$$

$$\hat{H} = \frac{\hat{\boldsymbol{p}}^2}{2m} + \hat{V}(\boldsymbol{r}) = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \hat{V}(\boldsymbol{r}).$$ "Hamiltonian"

The position operator is given by

$$\hat{\boldsymbol{r}} = \boldsymbol{r},$$

the momentum operator is:

$$\hat{\boldsymbol{p}} = -\mathrm{i}\hbar\boldsymbol{\nabla}.$$

$$\Psi(\boldsymbol{r},t) = \psi(\boldsymbol{r})\,\Theta(t). \quad \Psi(\boldsymbol{r},t) = \psi(\boldsymbol{r})\exp(-\mathrm{i}Et/\hbar).$$

time-independent Schrödinger equation:

$$\hat{H}\psi(\boldsymbol{r}) \equiv -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi(\boldsymbol{r}) + \hat{V}(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}).$$

Several Should Know Points

- Example: free particle: $\hat{H}\psi_n(\boldsymbol{r}) = E_n\psi_n(\boldsymbol{r}),$
- Probability density: $\int_{-\infty}^{+\infty} |\psi(x)|^2\,\mathrm{d}x = 1,$
- Normalizable (bound) vs unnormalizable (extended) state
- Superposition state $\psi = \sum_i c_i\varphi_i$
- Measurement(!..“weak”measure) & expectation value/ variance

$$(\Delta O)^2 = \int \psi^*(\hat{O} - \langle\hat{O}\rangle)^2\psi\,\mathrm{d}^3\boldsymbol{r}$$
$$= \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2.$$

- Commutator/commutation & general uncertainty principle

$$[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}. \qquad \Delta A)^2(\Delta B)^2 \geq \left|[\hat{A},\hat{B}]\right|^2/4,$$

$$[\hat{x},\hat{p}_x] = \mathrm{i}\hbar. \qquad \Delta x\Delta p_x \geq \hbar/2.$$

ii) Matrix formulation (Heisenberg)

iii) Dirac “bra-ket” … quantum state/Hilbert space

“Hamiltonian” [“*Abrahams example”*] $\hat{H} = \hat{T} + \hat{V},$

Wave function <-> vector in Hilbert space

Particle in a box/1D Schrodinger equation

- Infinite square well (uncertainty)
- Finite square well
- 1D scattering state (refection & transmission --- “quantum reflection” & tunneling, even “resonant tunneling”)

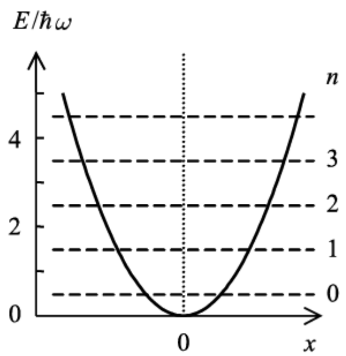
Harmonic oscillator

- Harmonic trap
- Quantized EM fields…

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2}+\frac{1}{2}m\omega^2x^2\psi(x)=E\psi(x).$$

$$\psi_n(x)=u_n(x)\exp(-m\omega x^2/2\hbar),$$

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega,$$



n	$u_n(x)$
0	$(1/a\sqrt{\pi})^{1/2}$
1	$(1/2a\sqrt{\pi})^{1/2}\,2(x/a)$
2	$(1/8a\sqrt{\pi})^{1/2}\,[2-4(x/a)^2]$
3	$(1/48a\sqrt{\pi})^{1/2}\,[12(x/a)-8(x/a)^3]$
$\vdots$	
n	$(1/n!2^n a\sqrt{\pi})^{1/2}\,H_n(x/a)$

Harmonic length $a=(\hbar/m\omega)^{1/2}$

for the nth level (see Exercise 3.11):

$$\Delta x\Delta p_x=\left(n+\frac{1}{2}\right)\hbar.$$

Lecture3 notes are collected by Bohao Liu. Correction, supplement and suggestion are greatly welcomed. Bohao Liu's email is liubohao@purdue.edu.