Entanglement

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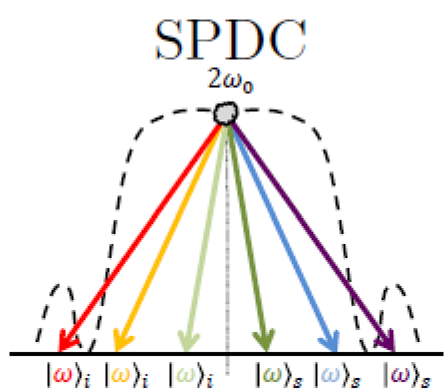
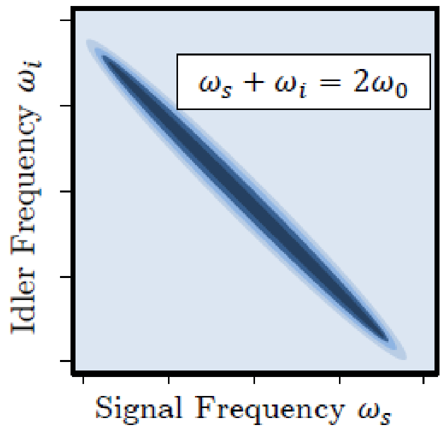
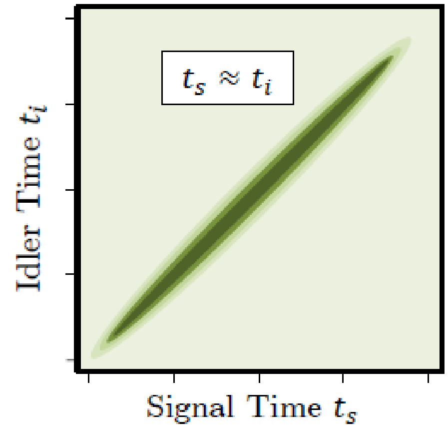
Lecture Notes 16, ECE695 Introduction to Quantum Optics and Quantum Photonics
Prof. Yong P. Chen, March 26, 2014

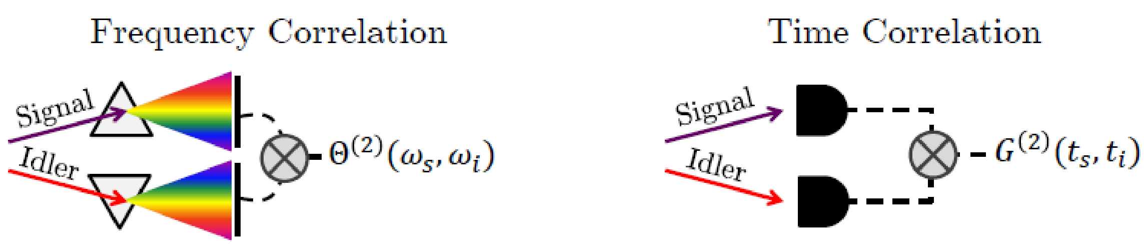
Entanglement is about two highly correlated particles or photons in the way that if we measure one of the particle's or photon's characteristics, we could have information about the other one's with high probability. For example, if we shine a beam with frequency $2\omega_0$ into a nonlinear crystal which causes "Spontaneous Parametric Down Conversion", some photons from this beam break into two correlated photons with center frequency of ω_0. We name the photon with higher frequency "signal" and the one with the frequency lower than ω_0 "idler". These two photons are the ones that we call entangled photons or "biphotons". We could say that These two photons are time-frequency correlated which means if we measure one of the photon's arrival time/frequency, we know the other one's arrival time/frequency without using the other photon itself for measurement. This is the fundamental approach for Quantum Key Distribution. If we generate these biphotons and send them to two different parties, say Alice and Bob, if Alice measures her biphoton's state, she knows what Bob's biphoton state would be and therefore, they could build a code based on this information that no one in the else can have and therefore they could have a complete secure transmission using these states to code their data and send it to each other.

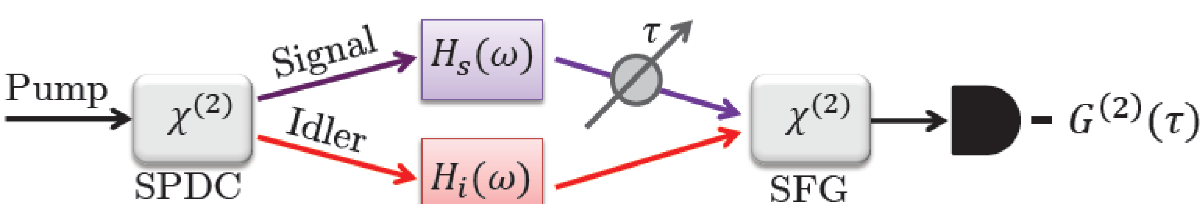


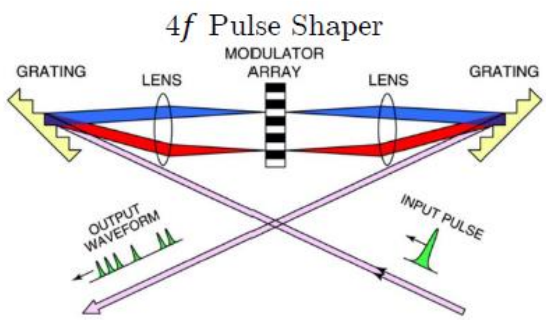


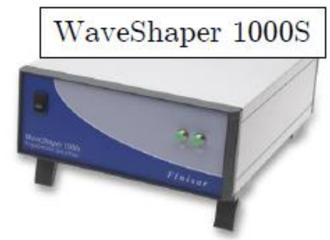


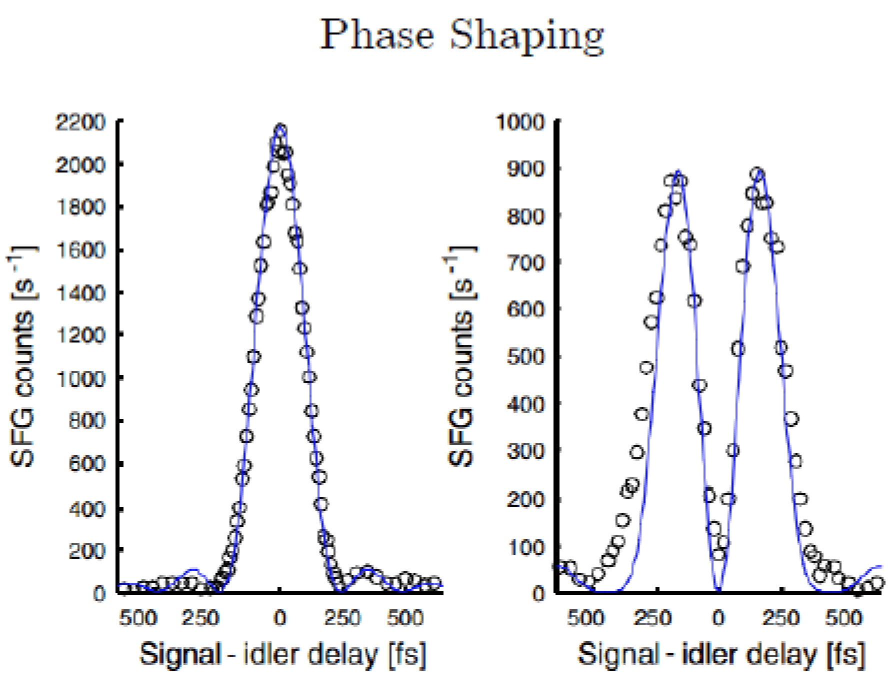
In experimental setups that we demonstrate in Prof. Andrew M. Weiner group's (Ultrafast Optics and Optical Fiber Communications Laboratory), we use two different paths for the signal and idler part and we use pulse shaper on one arm and then use another nonlinear crystal to measure these two arm correlations. This crystal does a different job and brings these two signal and idler photons and use Second Harmonic Generation or Sum Frequency Generation to rebuild the original photons and we estimate the correlation as a function of delay which we take into account on one arm with counting those photons with frequency $2\omega_0$ with a detector in the end.







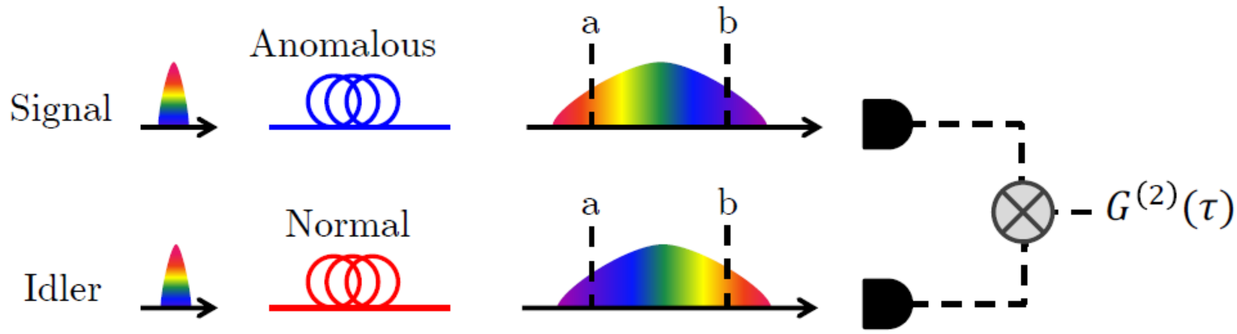




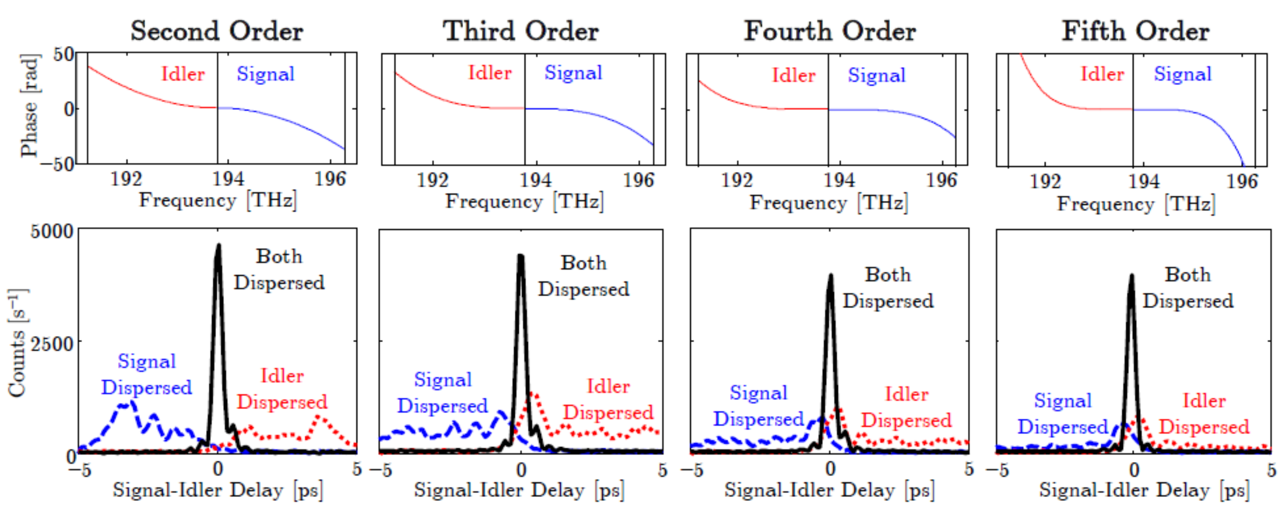
The figure above shows that the correlation between two arms becomes less when we apply the delay two one arm on the left figure, but if we use the wave shaper to shift half of the signal part's phase for 90 degrees in the frequency domain, we will get two peaks in our count curve.

Dispersion Compensation

In a fiber, different frequencies travel with different velocities because the refractive index of the fiber if depended on frequency. This causes dispersion which means out short pulses will be widened in time and if we have a transmitting data, out data rate should be less so that the pulses wouldn't interfere. We show that with this entangled photons we could introduce a regime to cancel the dispersion of two arms using different fibers on each arm. We have two different kinds of dispersion, anomalous dispersion in which higher frequencies travel faster and normal dispersion in which lower frequencies velocities are higher. If we use anomalous dispersion on one arm and normal dispersion on the other arm, in the end after Sum Frequency Generation, because each photon in the signal arm has a highly correlated photon in the idler arm, these dispersions cancel out and we could still have out ultrashort pulse in the end.

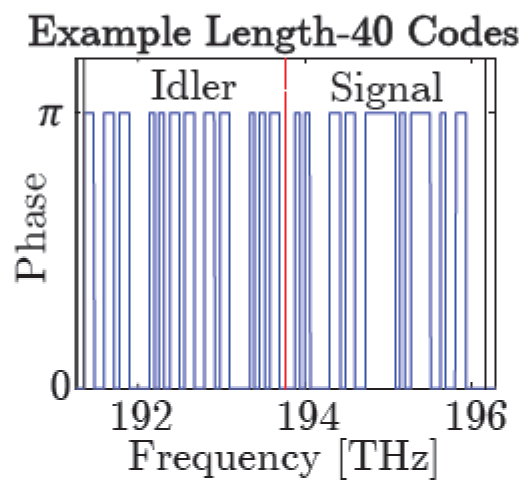


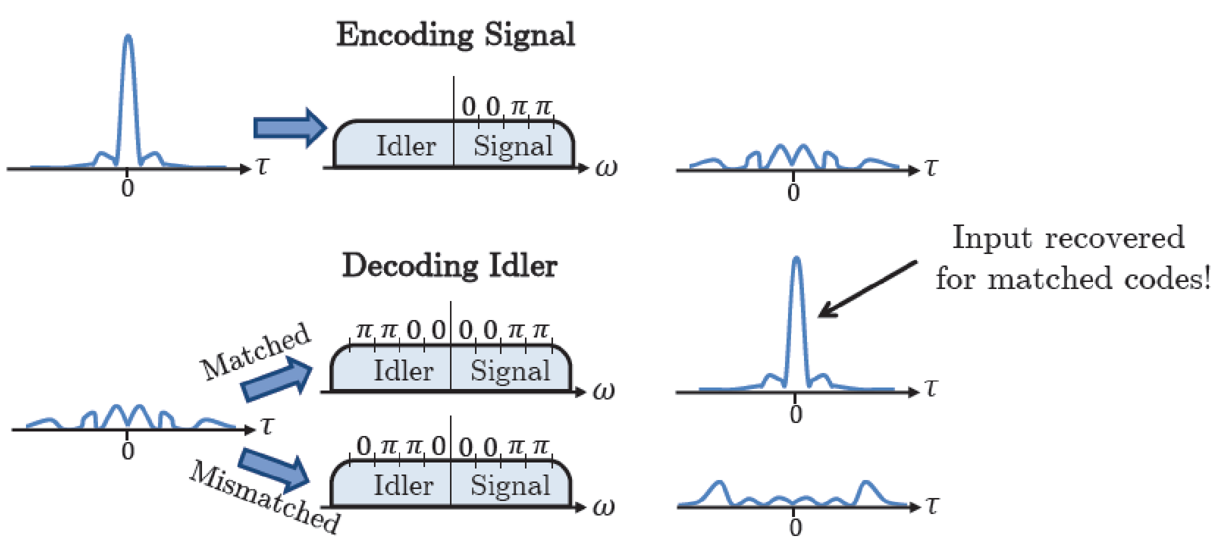
This approach only cancel out the second order dispersion but there are higher order dispersions which we could cancel out using a pulse shaper on each arm instead of regular anomalous and normal dispersion.



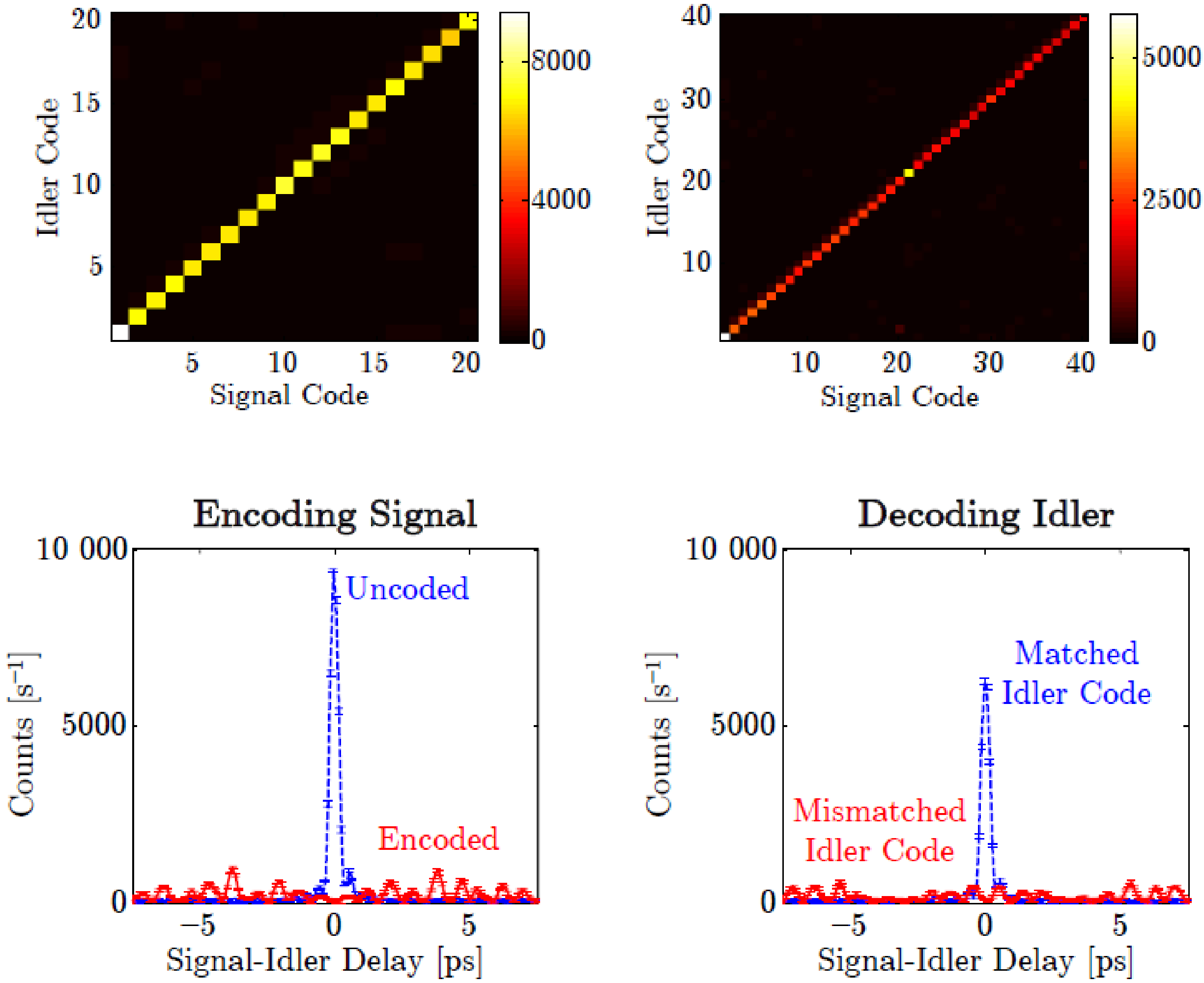
Orthogonal Spectral Coding

Because of the correlation between signal and idler, we could use Optical Code-Division Multiple Access (O-CDMA) on those. If we divide each arm's frequency domain into a number of smaller ones and use phase coding on each part, if the codes are matched on both arms, we could still have the correlation but because these codes are orthogonal, if one arm that in fact is a party in our transmission, uses another code for these slots phases, the correlation wouldn't happen.



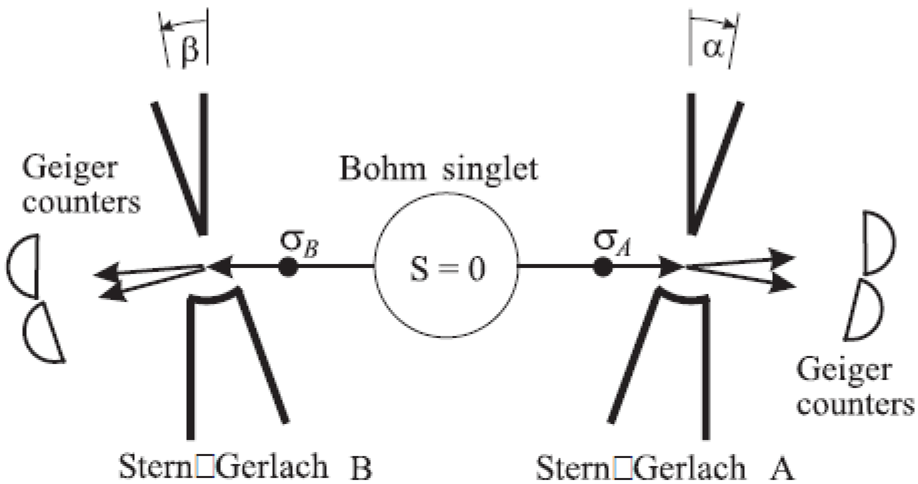


The experimental results for length-20 and length-40 are shown below. These results support our idea that these codes are completely orthogonal and we couldn't have the correlation function peak without using the exact similar codes in each arm.


Idler Code
Signal Code
Idler Code
Signal Code
Encoding Signal
Counts [s^{-1}]
Uncoded
Encoded
Signal-Idler Delay [ps]
Decoding Idler
Counts [s^{-1}]
Matched
Idler Code
Mismatched
Idler Code
Signal-Idler Delay [ps]

States of photons and mathematical aspects

The key notion to understand the entanglement is we should say that from the point that the two entangled particles or photons are generated until we measure one of their states, none of them has an exact state to stay on. We could only say before the measurement there is a probability function for each particle to be on a certain state, but the thing that fixes these two particles into two particular states is our measurement. We could say that if we make a measurement on one of these particles, we trap not only that particle in a particular state, but also it affects the other particle's state and does the same thing to that as well. In other words, the entanglement between these two particles "collapses" when we make a measurement on one of them and after that, if we do something on one particle, for example change the polarization, this will not affect the other one's state and will not change that.



We consider two different systems A and B which are in the states of i and j. An example for these systems is that A could be an electron and B could be a proton:

A	$\|+\rangle$	$\|-\rangle$	Complete set
B	$\|\uparrow\rangle$	$\|\downarrow\rangle$	Complete set

In this case, we could introduce these systems as:

$$|A\rangle = a_+|+\rangle + a_-|-\rangle$$

$$|B\rangle = b_\uparrow|\uparrow\rangle + a_\downarrow|\downarrow\rangle$$

Then we could introduce the states of compound system AB and its tensors:

$$|AB; ij\rangle = |A;i\rangle|B;j\rangle$$

$$\langle AB; ij| = \langle A; i|\langle B; j|$$

$$\langle AB; i'j'|AB; ij\rangle = \langle A; i'|A; i\rangle\langle B; j'|B; j\rangle$$

The probability of having the state of $i'j'$ will be:

$$P_{AB}(i'j') = |\langle AB; i'j'|AB; ij\rangle|^2 = P_A(i')P_B(j')$$

Utilizing the 2 2 systems introduced before, we could write down:

$$\begin{aligned}|AB\rangle &= |A\rangle|B\rangle = (a_+|+\rangle + a_-|-\rangle)(b_\uparrow|\uparrow\rangle + b_\downarrow|\downarrow\rangle)\\ &= a_+b_\uparrow|+\rangle|\uparrow\rangle + a_-b_\uparrow|-\rangle|\uparrow\rangle + a_+b_\downarrow|+\rangle|\downarrow\rangle + a_-b_\downarrow|-\rangle|\downarrow\rangle\end{aligned}$$

The probability of having $b \uparrow a +$ will be:

$$P(B \uparrow |A +) = \frac{P_{AB}(\uparrow +)}{P_A(+)}$$

But from Bays' Theorem we have:

$$P(B \uparrow |A +) = \frac{P_{AB}(\uparrow +)}{P_{AB}(\uparrow +) + P_{AB}(\downarrow +)} = \frac{1}{1 + \frac{P_{AB}(\downarrow +)}{P_{AB}(\uparrow +)}} = \frac{1}{1 + \frac{P_{AB}(\downarrow +)}{P_{AB}(\uparrow +)}} = \frac{1}{1 + \left|\frac{a_+b_\uparrow}{a_+b_\downarrow}\right|^2} = \frac{1}{1 + \left|\frac{b_\uparrow}{b_\downarrow}\right|^2}$$

This probability doesn't depend on which state system A has, so in the case that $|AB\rangle = |A\rangle|B\rangle$, we could say that the systems are not correlated.

Of course, we couldn't have the above equation for every compound system.

For two entangled particles, for example in polarization, we could draw these equations for the wave function:

$$|AB; ij\rangle = \sum_{ij} c_{ij}|A;i\rangle|B;j\rangle$$

$$c = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$c = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

In this case, the bell states for the horizontal and vertical polarizations will be:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|H\rangle_B - |V\rangle_A|V\rangle_B)$$

EPR Paradox

If we have a particle with spin-0 and it collapses into two entangled particles, from the conservation of momentum, we know that these two will emit in opposite directions and from conservation of angular momentum, we know that one of them has spin $+\frac{1}{2}$ and the other one has the spin $-\frac{1}{2}$. In this case, we could write the Bohm's singlet state as:

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$$

If we have a measurement S_x^A which is the spin measurement on x for A particle and it was $+\frac{1}{2}$ we could say that we know the S_x^B spin with certainty and it's $-\frac{1}{2}$.

$$S_x^A = 1/2 \Rightarrow |\Psi\rangle_{AB}^x = |\uparrow_x\rangle_A|\downarrow_x\rangle_B \Rightarrow S_x^B = -1/2$$

But if we want to measure the spin in y, we reach to the conclusion that S_x^B and S_y^B are non-commuting measurements which are:

$$[S_x^B, S_y^B] = i\hbar S_z^B \neq 0$$

Local Realism

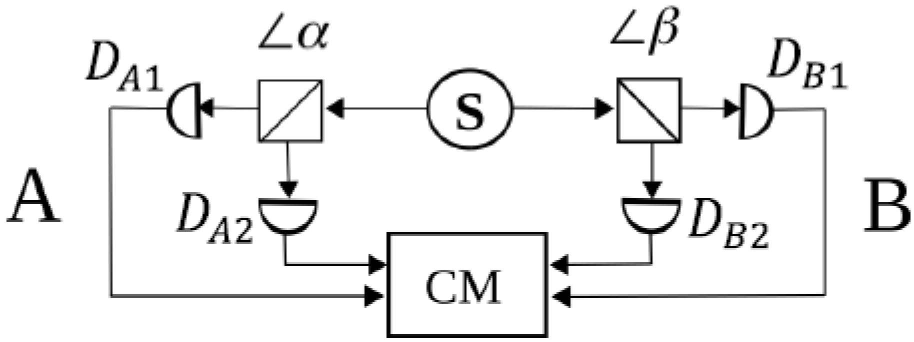
If we have a state space Λ with states λ, if:

1. Objective reality: Λ defined without reference to any measurement
2. Spatial separability: Λ_A Λ_B for spatially separated systems are independently defined

Λ supports probability distributions $\rho(\lambda)$

$$\rho(\lambda) \geq 0 \quad \int d\lambda \rho(\lambda) = 1$$

In this case, if we have a setup like the figure below, the detectors D_{A1} D_{A2} D_{B1} D_{B2} Outcome Parameters will be either +1 or -1. We introduce $p(D_{Am}|\lambda,\alpha,\beta) \equiv$ prob. of outcome D_{An} given the system state λ and parameter settings α and β.



From parameter independence and outcome independence we have:

$$p(D_{Am}|\lambda,\alpha,\beta) = p(D_{Am}|\lambda,\alpha)$$

$$p(D_{Bn}|\lambda,\alpha,\beta) = p(D_{Bn}|\lambda,\beta)$$

$$p(D_{Am}|\lambda,\alpha,\beta,D_{Bn}) = p(D_{Am}|\lambda,\alpha,\beta)$$

$$p(D_{Bn}|\lambda,\alpha,\beta,D_{An}) = p(D_{Bn}|\lambda,\alpha,\beta)$$

Then we have Bell's expectation values for A and B detectors (Alice and Bob) as:

$$E(\lambda,\alpha) = \sum_m p(D_{Am}|\lambda,\alpha)\, A_m$$

$$E(\lambda,\beta) = \sum_n p(D_{Bn}|\lambda,\beta)\, B_n$$

$$E(\lambda,\alpha,\beta) = \sum_{m,n} p(D_{Am},D_{Bn}|\lambda,\alpha,\beta)\, A_m B_n$$

But in absence of complete state information:

$$E(\alpha) \equiv \sum_m p(D_{Am}|\alpha)\, A_m$$

$$E(\beta) \equiv \sum_n p(D_{Bn}|\beta)\, B_n$$

$$E(\alpha,\beta) \equiv \sum_{m,n} p(D_{Am},D_{Bn}|\alpha,\beta)\, A_m B_n$$

If we have strong separabillity between α and β:

$$E(\alpha,\beta) = E(\alpha)E(\beta)$$

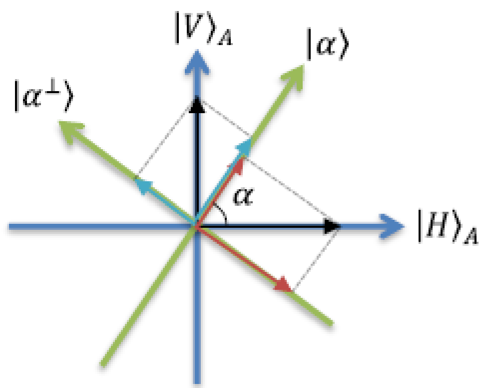
Then Bell's inequality is:

$$S \equiv E(\lambda,\alpha_1,\beta_1) + E(\lambda,\alpha_1,\beta_2) + E(\lambda,\alpha_2,\beta_1) - E(\lambda,\alpha_2,\beta_2)$$

And if λ is unknown:

$$S \equiv E(\alpha_1,\beta_1) + E(\alpha_1,\beta_2) + E(\alpha_2,\beta_1) - E(\alpha_2,\beta_2)$$

This parameter should be less than 2 but as we could see below, quantum mechanics violates Bell's inequality. For example if we choose $|\Psi^-\rangle$ and use two PBS with arbitrary angles α for A and β for B :



$|\alpha\rangle = \cos\alpha\,|H\rangle_A + \sin\alpha\,|V\rangle_A$

$|\alpha^\perp\rangle = -\sin\alpha\,|H\rangle_A + \cos\alpha\,|V\rangle_A$

$|\beta\rangle = \cos\beta\,|H\rangle_B + \sin\beta\,|V\rangle_B$

$|\beta^\perp\rangle = -\sin\beta\,|H\rangle_B + \cos\beta\,|V\rangle_B$

$|H\rangle_A = \cos\alpha\,|\alpha\rangle - \sin\alpha\,|\alpha^\perp\rangle$

$|V\rangle_A = \sin\alpha\,|\alpha\rangle + \cos\alpha\,|\alpha^\perp\rangle$

$|H\rangle_B = \cos\beta\,|\beta\rangle - \sin\beta\,|\beta^\perp\rangle$

$|V\rangle_B = \sin\beta\,|\beta\rangle + \cos\beta\,|\beta^\perp\rangle$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$$

$$= \frac{1}{\sqrt{2}}\big((\cos\alpha\,|\alpha\rangle - \sin\alpha\,|\alpha^\perp\rangle)\cdot(\sin\beta\,|\beta\rangle + \cos\beta\,|\beta^\perp\rangle) - (\sin\alpha\,|\alpha\rangle + \cos\alpha\,|\alpha^\perp\rangle)$$
$$\cdot(\cos\beta\,|\beta\rangle - \sin\beta\,|\beta^\perp\rangle)\big)$$

$$= \frac{1}{\sqrt{2}}(\cos\alpha\,|\alpha\rangle\sin\beta\,|\beta\rangle + \cos\alpha\,|\alpha\rangle\cos\beta\,|\beta^\perp\rangle - \sin\alpha\,|\alpha^\perp\rangle\sin\beta\,|\beta\rangle - \sin\alpha\,|\alpha^\perp\rangle\cos\beta\,|\beta^\perp\rangle$$
$$- \sin\alpha\,|\alpha\rangle\cos\beta\,|\beta\rangle + \sin\alpha\,|\alpha\rangle\sin\beta\,|\beta^\perp\rangle - \cos\alpha\,|\alpha^\perp\rangle\cos\beta\,|\beta\rangle$$
$$+ \cos\alpha\,|\alpha^\perp\rangle\sin\beta\,|\beta^\perp\rangle)$$

$$= \frac{1}{\sqrt{2}}\big((\cos\alpha\sin\beta - \sin\alpha\cos\beta)|\alpha\rangle|\beta\rangle + (\cos\alpha\cos\beta + \sin\alpha\sin\beta)|\alpha\rangle|\beta^\perp\rangle$$
$$- (\sin\alpha\sin\beta + \cos\alpha\cos\beta)|\alpha^\perp\rangle|\beta\rangle - (\sin\alpha\cos\beta - \cos\alpha\sin\beta)|\alpha^\perp\rangle|\beta^\perp\rangle\big)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(-\sin(\alpha-\beta)\,|\alpha\rangle|\beta\rangle + \cos(\alpha-\beta)\,|\alpha\rangle|\beta^\perp\rangle - \cos(\alpha-\beta)\,|\alpha^\perp\rangle|\beta\rangle - \sin(\alpha-\beta)\,|\alpha^\perp\rangle|\beta^\perp\rangle)$$

$$E(\alpha,\beta) = \sin^2(\alpha-\beta) - \cos^2(\alpha-\beta)$$

$$E(\alpha,\beta) = -\cos(2\alpha - 2\beta)$$

$$\alpha_1 = 0^o,\ \ \alpha_2 = 45^o,\ \ \beta_1 = 22.5^o,\ \ \beta_2 = -22.5^o$$

$$E(\alpha_1,\beta_1) = -\frac{1}{\sqrt{2}} \qquad E(\alpha_1,\beta_2) = -\frac{1}{\sqrt{2}}$$

$$E(\alpha_2,\beta_1) = -\frac{1}{\sqrt{2}} \qquad E(\alpha_2,\beta_2) = +\frac{1}{\sqrt{2}}$$

$$S = 2\sqrt{2}$$

As we can see, S in this example is greater that two which violates Bell's inequality.