Analysis of shot noise in the detection of ultrashort optical pulse trains [2]

Poolad Imany

Abstract- in this report we are going to analyze the shot noise effect on the photocurrent output of the photo detector. We introduce amplitude and phase detection utilizing separate modulations and we are willing to lower the phase noise by "pushing" it to the amplitude and higher the amplitude noise. With this method, we will be able to use the phase as a low noise aspect in the photo detector and we could use it as the timing jitter. We also introduce Secant Hyperbolic pulses as the shape with the least phase noise which the other papers did not consider.

1. Shot Noise [1,4]

We know that every light beam consists of a lot of energy packets called photons. The number of photons in a high power light source is enormous in a way that we could neglect the discreteness of the light beam and consider it as an analog source. However, if we have a low power source, this discreteness becomes important and each photon takes into account. In this case, we are considering the power in the photo detector as the number of photons that it counts and as we know the energy of each photon is a particular number which depends on its frequency:

$$E = h\vartheta \quad (1)$$

Where h is the Planck's constant and ϑ is the frequency of the photon. The origin of the shot noise is, we don't know the exact number of the photon that the light source releases and we also don't know the exact position of the photons. In other words, we couldn't know the exact number of photons that the photo detector gets for an optical pulse and we don't know the exact time that each photon hits the photo detector. So, what we can say is that the photons have a probability density and we have the mean and variance of it, and we can only calculate the probability of having k photons for an optical pulse. For a thermal source, the distribution for having a certain number of photons is a Bose-Einstein

distribution and for lasers it will be the Poisson distribution. In case that we consider a white Gaussian channel noise as well, the distribution at the photo detector will be Laguerre distribution. In this report, we will use mode-lock lasers without channel noise so the distribution for our work will be Poisson:

$$P(k) = \frac{e^{-a}a^{k}}{k!} \quad (2)$$

Where a is the mean and also the variance of the distribution and k is the number of photons. The aspect of shot noise is because of this probability density [10].

We can see that in the shot noise situation, this is the signal itself that has a noise inside of it. For the classical transmissions, we had noisy channels, detectors, etc. but in this case if everything is ideal we still have the shot noise because of the uncertainty that exists inside of the photon origins.

2. The Overall Process

In this report, we have an optical pulse train which will be produced from a mode-locked laser and hits a photo detector with η efficiency. The light that we use in this report is not a squeezed light so we could say that each photon's arrival time to the photo detector is independent from other photons and the photon distribution is Poisson. The photo detector has an impulse response which should be causal so it is one sided as Fig.1 shows. Each of this impulse responses are related to releasing a single photoelectron so we could write:

$$\int h(t)dt = q \quad (3)$$

The current that the photo detector produces goes through two different processes for each individual frequency to calculate the amplitude and phase. We are willing to show that we could lower the phase noise caused by the shot noise by shortening the pulse and push it through the amplitude noise. For this process we should use the correlations between various frequencies for the output pulse. We will show that the upper frequencies and lower frequencies for each particular frequency are correlated and we use it to draw a conclusion about lowering the noise in the phase measurements so that we could use the phase as timing jitter between the transmitter and the receiver.

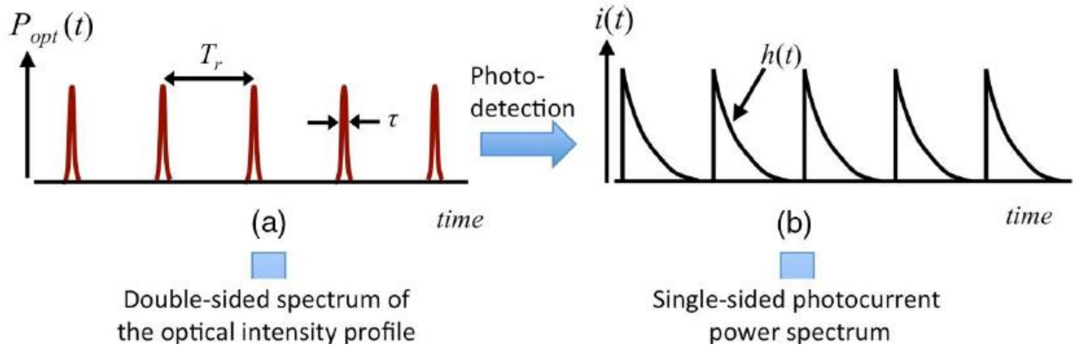

Fig.1. (a) Optical pulse train intensity profile. (b) Photo-detected electrical pulse train when the optical pulse width is much shorter than the photo detector's impulse response.

3. Semi classical Photocurrent Shot Noise Analysis [2,3]

Because we are not using the squeezed states of the transmitted beam, we could use below definition for the photocurrent:

$$i(t) = \sum_k X_k h(t - k\Delta t) \quad (4)$$

Here, $h(t)$ is the impulse response of the photo detector. We break each timing duration estimated for having all of each pulse's photons in it to k time slots with duration Δt . Δt should be small enough so we could say that in each time slot, we have either zero or one photon and we could neglect the possibility of having two or more photons in one time slot. In this case we could indicate that X_k is zero or one depended on if there is a photon in the K^{th} time slot or not. Based on previous equations, we could say that the probability of a photo-detection in Δt is:

$$p = \frac{\eta}{h\vartheta} P_{opt}(t)\Delta t = \lambda(t)\Delta t \quad (5)$$

In which $\lambda(t)$ is the photoelectron generation rate and the optical power will be:

$$P_{opt}(t) = P_0 \sum_n f(t - nT_r) \quad (6)$$

In which $f(t)$ is the pulse shape and T_r is the pulse train period. We could say that the power spectral density of the optical pulse train will be also a pulse train and because of the impulse response of the photo detector, the power spectral density of the photocurrent will be like Fig.2.

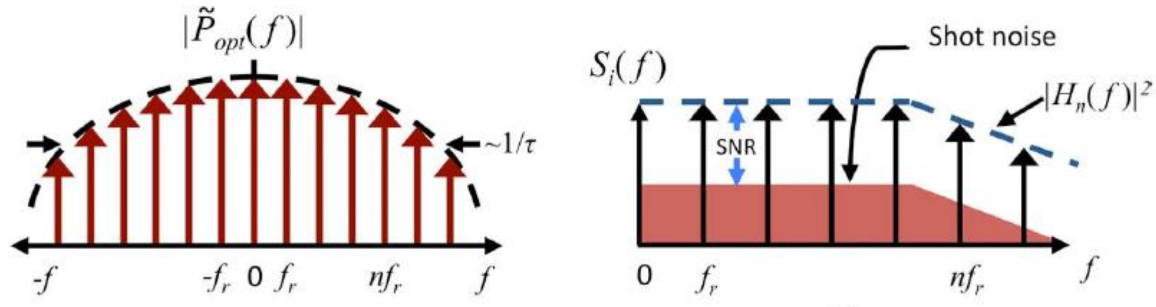

Fig. 2. The power spectral density of the optical pulse train and the power spectral density of the photocurrent

We could write the power spectral density of the photo current as:

$$S_i(f) = |H(f)|^2[\lambda_{avg} + S_\lambda(f)] \quad (7)$$

$$S_\lambda(f) = \left(\frac{\eta}{h\vartheta}\right)^2 \lim_{T\to\infty}[\left|F_T\{P_{opt}(t)\}\right|^2] \quad (8)$$

Where S_λ is the power spectral density of the photoelectron generation is rate and λ_{avg} is the average photoelectron generation rate. $H(f)$ is the Fourier transform of $h(t)$.

4. Noise Correlations

In this part we want to show that there are specific correlations between some particular frequencies in the optical power spectrum and the photocurrent spectrum as well. We will use the definition of photo current cross-spectral density for calculating the correlations:

$$S_i(f,f') = \lim_{T\to\infty}\frac{1}{T}[F_T\{\textstyle\sum_k X_k h(t-k\Delta t)\}.\ F_T^*\{\textstyle\sum_l X_l h(t-l\Delta t)\}] \quad (9)$$

If we use the Poisson distribution for the pulses and independency between the detected photons:

$$X_k X_l = \begin{cases} \gamma(k\Delta t)\Delta t & for\ k = l \\ \gamma(k\Delta t)\gamma(l\Delta t)\Delta t\Delta t & for\ k \neq l \end{cases} \quad (10)$$

Then the photocurrent cross-spectral density will be:

$$S_i(f,f') = H(f)H^*(f')\lim_{T\to\infty}\frac{1}{T}[\Lambda_T(f-f')] \quad (11)$$

Where $\Lambda(f)$ is the Fourier transform of $\lambda(t)$. If we have a CW light source, the cross-spectral density is non-zero just for $f = f'$ because the frequency content of a coherent CW signal is restricted to DC. However, for a pulse train, we could see from Fig.2 that if $f - f' = nf_r$ we will have non-zero cross spectral density for the photocurrent as well. So, the $mf_r + \delta f$ and $kf_r + \delta f$ frequencies in the photocurrent power spectral density are correlated to each other. From this assumption, we use the fact that $nf_r + \delta f$ and $-nf_r + \delta f$ frequencies are correlated and the correlation amount is proportional to $\Lambda_T(2nf_r)$ according to equation (11). On the other hand, because $\lambda(t)$ is a real function, the frequencies $nf_r - \delta f$ and $-nf_r + \delta f$ are correlated as well, in fact they are the same as each other with a mirroring. From these explanations, we could see that the upper frequency and lower frequency of nf_r are correlated and the magnitude of the correlation is:

$$|C(nf_r + \delta f, nf_r - \delta f)| = |C(nf_r + \delta f, -nf_r + \delta f)| = \frac{|S_{i_n}(nf_r + \delta f, -nf_r + \delta f)|}{[S_i(nf_r + \delta f)S_i(-nf_r + \delta f)]^{\frac{1}{2}}} = \frac{|\tilde{P}_{opt}(2nf_r)|}{\tilde{P}_{opt}(0)} \quad (12)$$

Here $\tilde{P}_{opt}(f)$ is the Fourier transform of $P_{opt}(t)$. The fact that shot noise is correlated between upper and lower sidebands implies that shot noise does not contribute equally to the phase and amplitude quadratures of the signal. We will see that this correlation expression comes into play for calculating the amplitude and phase noise.

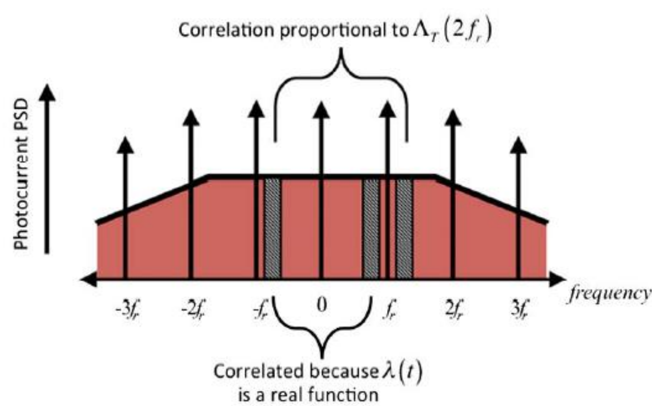

Fig.3. correlations in the sidebands in the photocurrent power spectral density

5. Amplitude and Phase Noise

Now we want to calculate the amplitude noise and phase noise for the shot-noise limited system. A regular way to measure these for a particular frequency is multiplying the signal with a cosine function with the same frequency and two different phases. First, we indicate that the photocurrent signal for the f_0 frequency will be:

$$[1 + a(t)].\cos\big(2\pi f_0 t + \theta(t)\big) \quad (13)$$

Where $a(t)$ is the amplitude noise and $\theta(t)$ is the phase noise. We then multiply it by:

$$\cos(2\pi f_0 t + \Phi_r) \quad (14)$$

We could see for $\Phi_r = 0$ and $\Phi_r = \frac{\pi}{2}$ we will have:

$$\Phi_r = 0: \quad output1(t) = \frac{1}{2}[1 + a(t)] \quad (14)$$

$$\Phi_r = \frac{\pi}{2}: \; output2(t) = \frac{1}{2}\theta(t) \qquad (15)$$

These equations are correct if we consider that $a(t)$ and $\theta(t)$ are very small so we could use Taylor expansion in our expressions. We could see that $output1$ express the amplitude noise if we eliminate the DC part and $output2$ is the assumption for the phase noise. We now use these methods on our photocurrent and from using these demodulations on it; we will calculate its amplitude and phase noise [8,11]. The thing that we should take into account is that for the detection of ultrashort pulses, we model amplitude and phase noise measurements as the isolation of the harmonic of the repetition rate with a bandpass filter, so the demodulated photocurrent will be:

$$i_m(t) = [(\sum_k X_k h(t - k\Delta t)) * g(t)].\cos(2\pi n f_r t + \Phi_r) \quad (16)$$

Where $g(t)$ is the bandpass filter. If we derive the spectral density for this demodulated current, we will see that the amplitude and phase noise will be:

$$L_{AM,PM} = \frac{qI_{avg}|H_n(nf_r)|^2 R}{P_\mu(nf_r)}\left[1 \pm \frac{|\tilde{P}_{opt}(2nf_r)|}{\tilde{P}_{opt}(0)}.\cos\left(2\Phi_{opt}(nf_r) - \Phi_{opt}(2nf_r)\right)\right] (Hz^{-1}) \quad (17)$$

Where R is the terminating load impedance and:

$$P_\mu(nf_r) = 2q^2|H(nf_r)|^2 S_\lambda(nf_r) R\Delta f \qquad (18)$$

Is the single-sided power for one harmonic in a narrow bandwidth Δf with nf_r center frequency. The plus sign in equation (17) is for the amplitude noise and the minus sign is for the phase noise. We can also see that the assumption $\frac{|\tilde{P}_{opt}(2nf_r)|}{\tilde{P}_{opt}(0)}$ is the correlation between upper and lower sidebands of nf_r except for the cosine term which is not important if we choose the phases carefully.

From equation (17) we could say that since for the long pulses the correlation disappear, the term in brackets could be considered as the deviation of shot noise. Here we will consider it for some special pulse shapes and analyze it.

Gaussian Pulses

In this part we consider a Gaussian pulse train which will be described as:

$$P_{opt}(t) = \frac{E_p}{\tau_G\sqrt{\pi}}\sum_n \exp\left\{-\left(\frac{t - nT_r}{\tau_G}\right)^2\right\} \qquad (19)$$

Where $\tau_G = \frac{\tau_p}{2\sqrt{2ln2}}$ and τ_p is the full width half max of the pulses. Using this expression and equation (17) gives us:

$$L_{AM,PM} = \frac{qI_{avg}|H_n(nf_r)|^2 R}{P_\mu(nf_r)}[1 \pm \exp\{-(2\pi nf_r\tau_G)^2\}] \qquad (20)$$

This is because we have either zero Φ_{opt} or we have that due to the arbitrary time delay which cases to $2\Phi_{opt}(nf_r) = \Phi_{opt}(2nf_r)$. The plots for amplitude and phase noise for 1st and 3rd harmonics are in Fig.4.

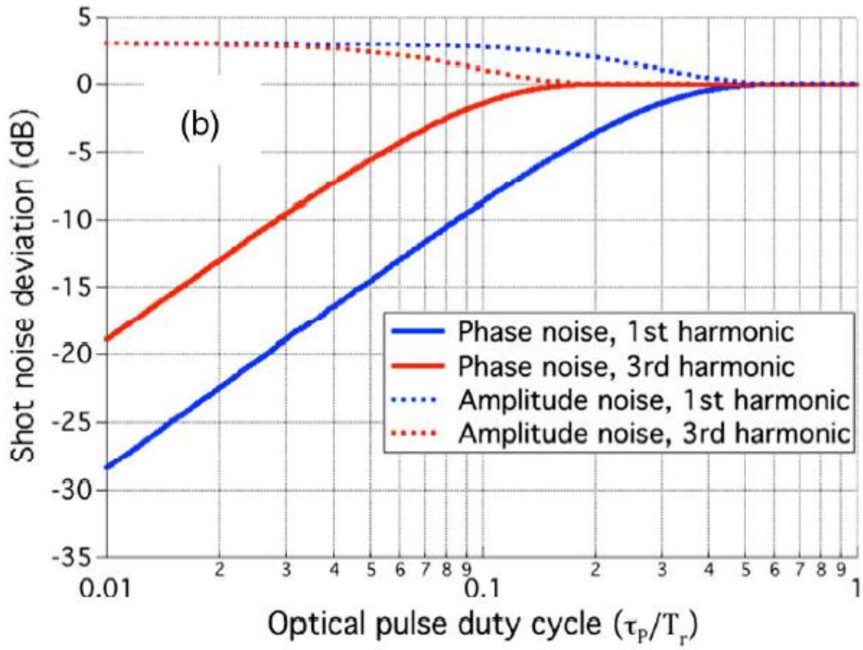

Fig.4. The amplitude and phase noise for the first and third harmonics of a Gaussian pulse train

We could say that if the pulse is too long, we could estimate the amplitude and phase noise with:

$$L_{AM,PM} = \frac{qI_{avg}|H_n(nf_r)|^2R}{P_\mu(nf_r)} \qquad (21)$$

If the pulses are short enough, with using equation (18), the amplitude and phase noise are approximately:

$$L_{AM} = \frac{q}{I_{avg}} \qquad\qquad L_{PM} = \frac{q}{2I_{avg}}(2\pi nf_r\tau_G)^2 \qquad (22)$$

From the above equation, we could see that when the pulses are too short, we can lower the phase noise as low as we could shorten the pulse so we could have perfect phase detection and we could use it as a timing jitter between the source and the detector [9,12]. From Fig.4, we could see that at some points, when we are lowering

the pulse width, we are practically push the phase noise into the amplitude noise and this is the aspect of squeezing that we are using in this report.

6. Comparison Between Various Pulse Shapes

In this part, we calculate the phase noise for ultrashort pulses with different shapes (Gaussian, Square and Sech) and compare the results with each other. Neglecting the optical phases, the phase noise for a square pulse train and a sech pulse train will be:

$$Square\ pulse:\ \ L_{AM,PM} = \frac{qI_{avg}|H_n(nf_r)|^2 R}{P_\mu(nf_r)}\left[1 \pm \left|sinc\left(2\pi nf_r\tau_p\right)\right|\right] \quad (23)$$

$$Sech\ pulse:\ \ L_{AM,PM} = \frac{qI_{avg}|H_n(nf_r)|^2 R}{P_\mu(nf_r)}\left[1 \pm \mathrm{sech}(2\pi \mathrm{nf_r}\tau_s)\right] \quad (24)$$

If $\tau_G = \tau_p = \tau_s$ for these three kinds of pulses, if the pulse width is too short we could see that the phase noise between these three has a linear relation and Sech pulse has the minimum phase noise.

$$L_{PM,Gaussian} = 6L_{PM,Square} = 8L_{PM,Sech} \qquad (24)$$

In the case that all of these three has the same full width half max, we could see the phase noise for first and third harmonics in Fig.5 and Fig.6. As Fig.5 and Fig.6 show, the Sech pulses has the minimum phase noise again so I think it is reasonable if we use Sech pulses in experiments instead of Gaussian pulses which were used.

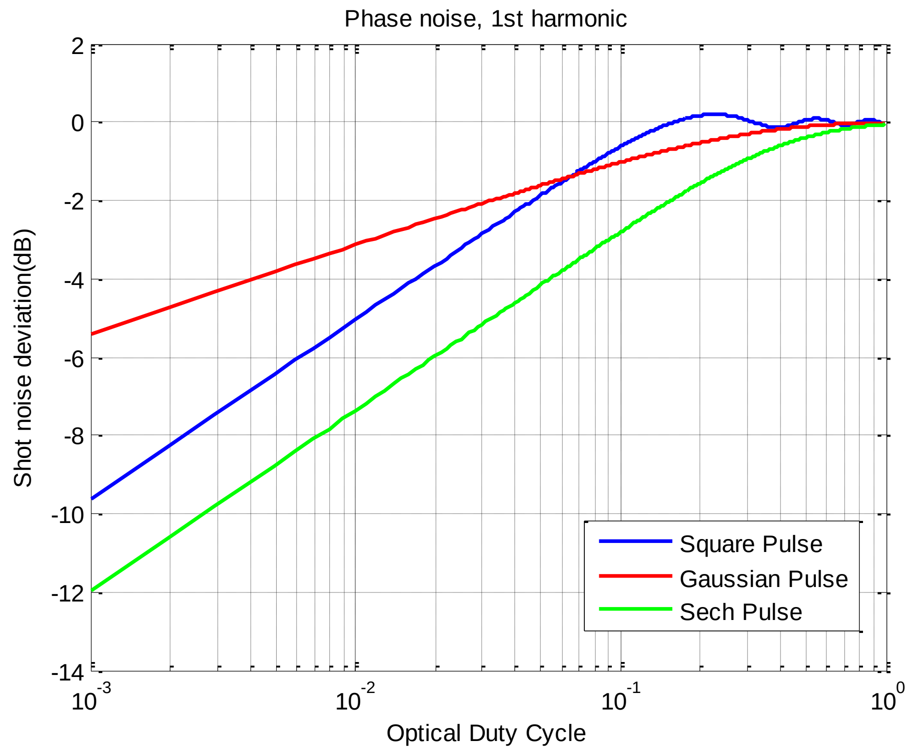

Fig.5. Phase noise for Gaussian, Square and Sech pulse trains for the first harmonic

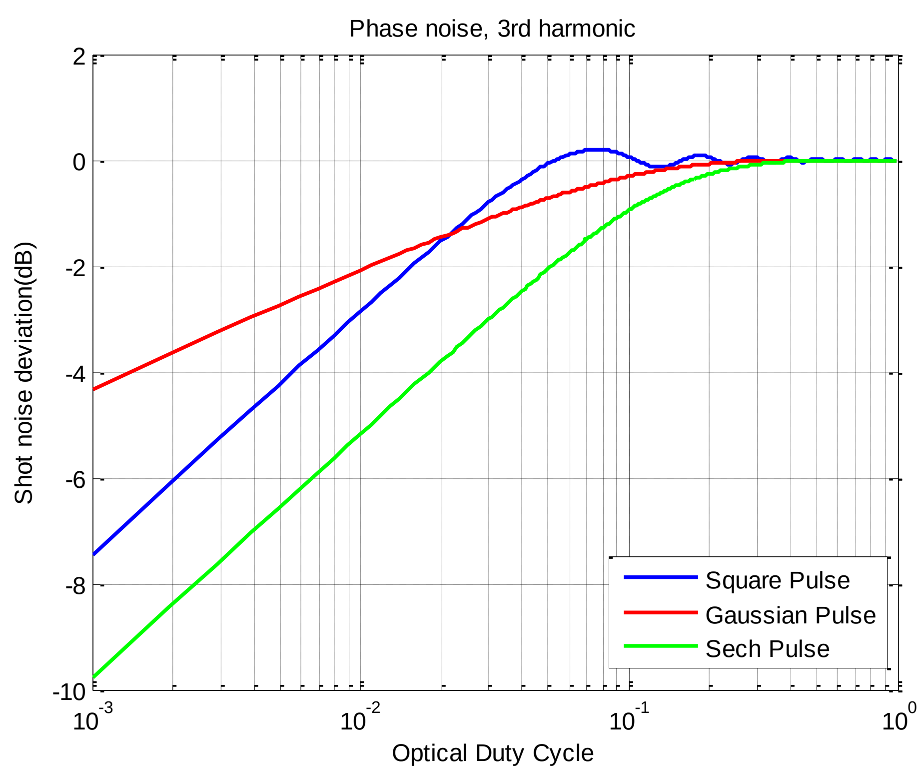

Fig.6. Phase noise for Gaussian, Square and Sech pulse trains for the third harmonic

7. Squeezing Aspect of the Work

In the squeezing light, the electric field uncertainty will be pushed through amplitude to phase and reverse. This aspect will achieve through the uncertainty of position and momentum for each photon. The correlation between these two allows us to reduce one's uncertainty (noise) and push it through the other one [7]. Fig.7 shows this thing if we consider $X_1 = E_{real}$ and $X_2 = E_{imaginary}$.

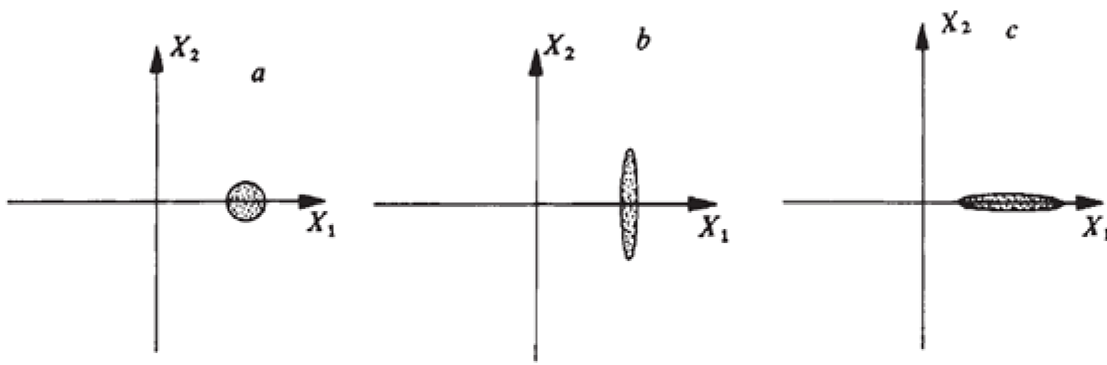

Fig.7. the uncertainty which is pushed from the real part of the electric field to the imaginary part

If the light is squeezed, the distribution of the detecting photons will not remain Poisson, so each photon arrival time won't be independent of the other photons [5,6]. In this case, we cannot do the noise pushing from the phase to the amplitude but I think we could do that by some kind of other demodulations which cancels out the dependency of the photons. I think this could be a good topic for further research.

8. Conclusion

In This report, we showed that for a mode locked laser with un-squeezed light source, we could push the phase noise into the amplitude noise after observing the photocurrent which is caused by a photo detector. This approach will be achieved by shortening the pulse and as we showed, the shorter the pulse is, the lower the phase noise will be. From using this aspect, we could lower the phase noise until we could use the phase detection as a timing jitter.

9. References

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